NATURAL PHILOSOPHY FOR BEGINNERS.

NATURAL PHILOSOPHY

FOR

BEGINNERS.

SECOND EDITION, IMPROVED.

NATURAL PHILOSOPHY

FOR BEGINNERS.

BEING FAMILIAR ILLUSTRATIONS

OF THE

LAWS OF MOTION AND MECHANICS.

INTENDED

AS A TEXT-BOOK FOR SCHOOLS AND SELF-INSTRUCTION;
AS A COMPANION TO THE LECTURE-ROOM,
OR FOR MODEL-SCHOOLS.

ILLUSTRATED BY ENGRAVINGS.

SECOND EDITION, REVISED AND IMPROVED.

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ADVERTISEMENT

TO

THE SECOND EDITION.

In this Edition many important corrections have been made, and some parts re-written. The arrangement adopted is the same as that of the first edition; as it was thought, that any departure from the original plan might occasion inconvenience in those schools in which the first edition has hitherto been used.

May 1845.

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INTRODUCTION.

The study of Natural Philosophy has of late years been so generally applied to the practical concerns of life, that it is now considered as an essential branch of education; but besides the advantage derived from a knowledge of the many interesting facts connected with its practical application, it has the still greater advantage of being the means of disciplining the mental faculties by creating habits of attention and correctness, and by combining an exercise of the understanding with that of the memory. Nor should the influence of this study, in a moral point of view, be lost sight of; for it produces a sincere and disinterested love of truth, destroys the effects of prejudice, and if properly directed, cannot but increase our admiration of the wisdom, the goodness, and the power of the Creator; and although proofs of the attributes of the Deity abound in every direction, yet, from the very circumstance of their being so abundant, the mind is apt to become callous to their beauties. "Now Science," says the Rev. H. Moseley, "opens to us on these points *new* views infinitely more striking than any that can be seen by the untutored intellect; views calculated to impose gratitude on the most insensible, and to bend in worship the minds of the most stubborn."

Of all the departments of science there is, perhaps, none of more practical utility than Mechanics, or from which mankind have derived greater advantages: a knowledge of this science is one of those things that serves to distinguish civilized nations from savage; for were man ignorant of the various arts which depend upon the Mechanical Powers, he would be but little superior in point of enjoyment to the beasts of the forest, and much their inferior in point of security. From this science the works of art derive much of their beauty and value: by its aid we are enabled to improve every power and force in nature, and the motions of the elements, water and air, are rendered subservient to the various purposes of life. As a branch of early instruction, there is perhaps no kind of study better adapted to the taste and capacity of youth, or more replete with utility and entertainment. "Every-body talks of the lever, the wedge, and the pulley; but most people perceive that the notions which they have of their respective uses are unsatisfactory and indistinct; and many endeavour at a late period of life to acquire a scientific and exact knowledge of the effects that are produced by implements which are in everybody's hands, or that are absolutely necessary in the daily occupation of mankind*."

The mental advantage derived from a practical application of the knowledge acquired in the schoolroom to the common affairs of life, need scarcely be suggested to the tutor; Mechanical Science affords numerous opportunities for thus exercising the ingenuity of boys. The apparatus necessary to "collect specimens" in this branch of science is not costly, being nothing more than an ordinary exercise of observation and attention; nor do the "specimens" so collected need any museum or glass case, as the only storehouse required is that of the pupil's mind. If the student has a desire to apply the knowledge he possesses to what daily passes before his eyes, he cannot take an hour's walk without meeting with some illustrations of this science. A visit to a factory is rendered doubly interesting by a knowledge of the principles of Machinery; nor will an intelligent boy consider his tutor a bore even in the play-ground if a principle of science be pointed out to him while playing at trap-ball or ring-taw. It was by observing trivial effects and studiously endeavouring to trace these effects to their cause, and to classify them under acknowledged principles, that Sir Isaac Newton was led to the discovery of universal gravitation; for upon

* Edgeworth's Practical Education.

observing an apple fall to the ground, he thought it possible that the same law which caused its descent might also keep the planets in their respective positions, and this he found to be the case. The laws of the pendulum were discovered by Galileo merely from his observing a lamp swinging from the ceiling.

Although numerous elementary works on the Natural Sciences have been published of late years, many of which have been deservedly popular (such, for instance, as Mrs. Marcet's Conversations, and Joyce's Scientific Dialogues), it is, nevertheless, generally admitted that these works are more adapted for home education than for class-books in schools. The present elementary treatise is an humble attempt to supply what the author deemed a desideratum,—viz. a work suitable for those commencing Natural Philosophy in Schools, devoid, on the one hand, of Mathematical formulæ, and requiring, on the other, rather more exercise of the intellect than is necessary for the comprehension of popular introductions similar to those above named. In compiling the present work, the Author's aim has chiefly been to explain the principles of Mechanical Science with precision and simplicity, and to illustrate them with apt and interesting examples; the demonstrations require no further knowledge of Mathematics than the elementary rules of Arithmetic, and care has been taken that the learner should meet with no technical difficulty in his progress. It was originally intended to accompany a set of apparatus for teaching Mechanics, sold by Messrs. Taylor and Walton, London; and with this idea a considerable number of the diagrams were copied from the set, but during the progress of the work it was found necessary to introduce many other examples, which, although in many cases illustrative of the same principles, were not deemed necessary to be included in a set of Mechanical Models, as they could be readily understood by attention to diagrams. But the work is not confined to the explanation of the Models above named: it may be used with any other set of Mechanical Apparatus, or even where the Pupil has not the advantage of Models; and the Author hopes it will be found a useful compendium for Model Schools, and Mechanics' Institutions, as he has been careful to embrace all the leading principles of the Science, and to render the subject as entertaining as he was able to the opening mind.

To those who possess a knowledge of algebraical formulæ, an attentive study of the works of Whewell, Gregory, and Pratt, in English, and Poisson and Poinsot in French, is recommended; and by those who wish to pursue the study of Mechanics further, without the aid of mathematical knowledge, the Lectures of

Dr Young*, the Treatises in the Library of Useful Knowledge, in the Cabinet Cyclopedia, and Dr Arnott's Physics, will be found well worthy of a careful perusal. To the three latter works the Author has to acknowledge himself indebted for some interesting examples and diagrams, as well as to a work by Professor Chevallier of Durham.

The Author cannot conclude these prefatory remarks without expressing his sincere thanks to the Rev. W. Cook, of University College, for his kindness in looking over the work before publication, and for his many useful suggestions.

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* A new edition of this work, completing the different subjects to the present time, edited by the Rev. P. Kelland, M.A. and Thomas Webster, M.A. is in course of publication.

MECHANICS.

CHAPTER I.

MECHANICS may be considered as the basis or groundwork of all the other Natural Sciences, as its principles are founded on the laws of Motion, and without a knowledge of these laws it will be impossible to understand the effects or calculate the consequences of the motions affecting solids and fluids.

The term Mechanics is derived from the Greek word Μηχανή, signifying a *Machine*. It is usual in mathematical treatises to divide this science into two parts, called *Statics* and *Dynamics*. Statics (from the Greek word Στάτικος, stopping) treats of the powers which preserve bodies in a state of rest; and Dynamics (from the Greek word Δύναμις, power or force) relates to the causes of movement, or the forces producing motion. In the present elementary work these two branches will be explained in conjunction, as the difference between the states of a body at rest and a body in motion is merely the effect of a different mode of action of the same cause. It will therefore be necessary for the student to be well acquainted with the effects of motion and the gravitation of bodies previously to his entering upon the study of the Mechanical Powers.

ON MOTION AND ITS LAWS.

Motion is the act of a body changing its place; or the contrary of remaining at rest.

A body when in a state of rest is of itself unable to move, and when in motion it is also unable of itself to come to a state of rest; this quality of matter has been called *Inertia* (from a Latin word which implies inactivity). We know from experience that a rock on the surface of the globe does not change its position in respect to other things on the earth; it having of itself no power to move, would for ever remain at rest, unless moved by some external force. It is equally true that matter has no power to bring itself to rest when once put in motion; for having no life, either state must depend entirely upon external circumstances, matter being passive both to motion and to rest. That bodies continue in the state of motion or of rest in which they happen to be, so as to require force to change the state, will be seen from the following illustrations:

A horse on attempting to move a heavily-laden cart, has to overcome its *inertia;* but this once effected, he continues to draw the burden with ease which at first he could with difficulty move. On a stage-coach starting, the passengers are thrown backward, owing to their *inertia* opposing a resistance to their bodies acquiring at once the motion of the vehicle, and therefore tends to leave them behind; when the coach stops, the opposite result ensues. A man standing at the stern of a boat, if he is not careful, will fall into the water when the boat begins to move; and on the stopping of the boat, he will fall forward. A man jumping from a stage-coach at speed is in great danger of falling after his feet reach the ground, his body having acquired as much velocity as if he had been running with the speed of the coach. A person about to leap over a ditch commences running at some distance from it, that his body may acquire a motion and help him over. An equestrian standing on the saddle of a horse at full gallop, will step from it to the back of another horse galloping alongside at the same rate with as much ease as if the horses were standing still: the man has the same velocity with the horse that gallops under him, and keeps this velocity while he steps to the back of the other. If the other were standing still, the man would fly over his head; and if he were to step from the back of a horse that is standing still to the back of another galloping past him, he would be left behind. In the same manner a slack-wire dancer tosses several balls from hand to hand while the wire is in full swing; the ball, swinging along with the hand, retains the velocity, and when in the air follows the hand, and falls into it when it is in the opposite extremity of its swing. Likewise a ball dropped from the mast-head of a ship that is sailing briskly forward, falls at the foot of the mast, as it retains the motion which it had while in the hand of the person who dropped it, and follows the mast during its fall.

The property of *inertia* is simply shewn by the following experiment:—Balance a card upon the tip of the finger, and place a shilling on it; let the edge of the card be smartly struck, and it will dart off from beneath the shilling, but the shilling, by its inertia, will remain resting on the finger: this arises from the inertia of the shilling being greater than the friction of the card.

From these examples it will be readily seen that a body at rest would never move if force were not applied: we must not however infer that rest is the natural state of all bodies, because matter does not put itself in motion, as a few examples will shew that motion is as much the natural state of matter as rest, and that both rest and motion depend on the resistance or impulse of external causes; it may also be seen that there are great differences in the duration of motions, and that these differences arise chiefly from *friction* and the *resistance of the air*.

If a smooth ball be rolled along the ground it will soon stop, owing to the ground being rough; if it be rolled upon a smooth bowling green, it will continue longer in motion, because the impediments to its progress are less; but if it be rolled on a smooth and level sheet of ice, it hardly suffers any retardation from friction, and if the air be moving with it, will reach to a very considerable distance. A large spinning-top, with a fine hard point, set in motion in a vessel from which the air has been exhausted, will spin for a length of time, because the air offers no impediment to its motion. A pendulum set in motion in a vacuum will vibrate for a considerable time, having merely to overcome a slight friction at its point of suspension. Let a small brass wheel of uniform thickness be put in motion round a pin passing through its centre, let it be so regularly constructed that its different parts will just balance each other about its centre, so that its motion round the pin be neither increased nor diminished by the force of gravity*. The only causes, therefore, which retard its motion, are, the *friction* upon its axis, and the *resistance* of the air. If the friction be diminished by placing the axis itself upon friction wheels, its motion

* By the *force of gravity* is meant the tendency which all bodies have to fall towards the earth; this will be explained in the Chapter on Gravitation.

will be continued for a much longer time than when it revolves simply upon its axis. By placing the whole apparatus in the exhausted receiver of an air-pump the resistance of the air is removed; and then it is found to revolve with a velocity nearly uniform for a very considerable length of time.

From these examples it appears, that when a body is once put in motion by the agency of some force, the continuance and regularity of its motion is always increased as we diminish the number of impediments arising from friction, resistance of the air, and other retarding causes. As this is the case in every instance which comes under our notice, we conclude that if a body could be placed under such circumstances as to be entirely free from the operation of these causes, its motion would then become altogether uniform and perpetual. Any cause which moves a body when at rest, or changes the motion of a body already in motion, is called *Force:* it is often found convenient to represent forces by numbers, or lines drawn in the direction in which the forces act; the amount of force may at the same time be represented by the *length* of the line. Whatever opposes motion so as to retard the moving body, destroys its motion, or causes it to move in a contrary direction, is called *Resistance.*

Motion varies according to the manner in which the force acts. *Common* motion is when two or more bodies move together: a man standing on the deck of a ship when sailing has *common* motion with the ship.

Motion is said to be *Absolute* when a body is in motion with respect to a fixed object: a man walking from one place to another, or a ship sailing through the water, are examples of absolute motion.

Relative motion is when a body changes its position with respect to another body also in motion: thus, a man sitting in a coach moving along the road is *relatively* at rest, that is, with respect to the coach; but *absolutely* in motion, being moved with the coach from one place to another: he is also relatively at rest with respect to the other passengers in the coach, whether the coach be at rest or in motion. Where there has been an actual change of place in the common meaning of the term, the motion which produced it is termed *absolute* motion; whereas on the contrary, when the situation has been only relatively changed by an alteration in the position of surrounding bodies, it is said to be *relative*.

The *Velocity* of a body is the rate of speed at which it moves; and when a body is moving uniformly, its velocity is measured by the space or distance over which it moves, divided by the time spent in that motion: thus, if a body in three seconds, with an invariable motion, pass over thirty feet, its velocity is said to be ten feet per second.

Motion is said to be *uniform* when the moving body passes over equal spaces in equal times: if gradually increasing, it is said to be *accelerated;* and, if gradually decreasing, *retarded*.

Momentum is the force or power with which a body in motion strikes against another body, and it is measured by the quantity of matter or weight of the body multiplied by its velocity; therefore the force necessary to produce a certain degree of velocity to a ball weighing ten ounces, is five times as great as would produce the same effect on a ball at rest weighing only two ounces, for the larger ball moves with five times the force of the smaller ball, although the *velocity* is the same in both. But if the velocity of the smaller ball were to be augmented five times, the *momenta* or *quantities of motion* would then be equal; that of the larger ball being expressed by ten multiplied by one, and that of the smaller by two multiplied by five. It follows, therefore, that the *momentum* or *quantity of motion* in any body may be increased, either by increasing the quantity of matter moving with a certain velocity, or by keeping the same quantity of matter and increasing the velocity: if a man with a certain force throws from him a weight of fifty pounds to the distance of ten feet, he must apply twice the force to throw a weight of one hundred pounds to the same distance, or to throw the fifty-pound weight twice as far; but if he employs no more force than he did before, he will throw the hundred-pound weight only to the distance of five feet, and in that case the two bodies will have the same *momenta*, because $50 \times 10 = 100 \times 5$. Hence a small body may have as much motion as a large one, however disproportionate the bodies may be, provided that their velocities be reciprocally proportionable to their masses, that is, if the small body has as much more velocity than the large one, as it has less matter. It is for this reason that battering-rams have been disused in war since the invention of gunpowder; for a cannon-ball weighing thirty-six pounds, shot from a cannon, will produce the same effect as a battering-ram weighing fifty thousand pounds, provided the cannon-ball moves as many times swifter as it has less matter than the battering-ram.

CHAPTER II.

THE LAWS OF MOTION.

THE simplest principles to which all motions can be reduced are called the *Three Laws of Motion.*

The First Law of Motion.

Every body continues in its state of rest or of uniform motion in a straight line, unless compelled to change that state by forces impressed upon it.

It will be obvious to the mind of the student that this law depends upon what is called the *Inertia* of matter (already explained at page 2), by which it is to be understood not only that matter will not move without the application of some external force, but also that when once in motion it will maintain that state until it is stopped by some other force. Whenever, therefore, a body is put into motion, that motion would continue for ever in a straight line were it not stopped or impeded by some external force, such as the resistance of the air, the force of gravity, or friction; for the more these hindrances are diminished, the longer the motion continues, and we therefore conclude that if they were entirely removed the motion would never cease.

As a body moving freely cannot vary its velocity without a cause, neither can it alter its course without a cause; free motion, therefore, is *straight* as well as *uniform.* The simplest idea of straight motion is that of a bullet or arrow shot directly up or down.

A stone in a sling, the moment it is set at liberty, darts off in a straight direction, as an arrow from a bow, or a bullet from a gun. A body moving in a circle, or curve, is constrained to do what is contrary to its *inertia*, and consequently it must be acted upon by at least two forces. The force required to keep the body in the bent course is called *centripetal*, or centre-seeking force; while the *inertia* of the body, giving it a tendency to move outwards or in a straight line, is called the *centrifugal*, or centre-flying force.

In all cases centrifugal force tends to make bodies under its influence recede from a central point; and when it acts in conjunction with a centripetal force, the effect will be revolving motion: this is agreeable to the first law of motion, that every body in motion will continue to move on in a straight line with a uniform velocity unless another force act upon it.

The planets are illustrations of circular motion; the moon, for instance, has a constant tendency to the earth by the attraction of gravitation, and it has also a tendency to proceed in a straight line, and by the joint action of these two forces it describes a circular motion.

When a ball is shot from a cannon, it descends to the earth in a curved direction, which is caused by the attraction of gravitation; and, but for this force and the resistance of the atmosphere, it would continue to move onward in a straight line.

A tumbler filled with water and placed in a sling may be gradually made to revolve rapidly in a circle without a drop of the water being spilled; and even when the mouth of the glass is presented downward, the water will still be retained in it; as by its inertia, or centrifugal force, it tends more away from the centre of motion, and *towards* the bottom of the tumbler than towards the earth by gravity. A half-formed vessel of soft clay placed in the centre of the potter's table—which is made to whirl, and is called his wheel,—opens out or widens merely by the centrifugal force of its sides, and thus assists the workman in giving its form.

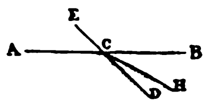
By reason of centrifugal force equestrians perform their feats of horsemanship in a small ring; both man and horse incline their bodies inwards to counteract the tendency which the centrifugal force has of impelling them outwards. Carriages are not unfrequently upset in turning a corner quickly: the body of the vehicle, owing to its inertia, has a tendency to move forwards, while the wheels are suddenly pulled round by the horses into a new direction.

The laws of motion are pleasingly exemplified in skating: a man when skating with great velocity will find himself obliged to lean considerably inwards on rounding a corner,—this gives rise to the variety of attitudes displayed by the expert skater.

Matter, owing to its *inertia*, is unable of itself to change the *direction* of motion. We have shewn above that if a body were impelled by a single force, and no resistance were made to it, it would continue to move onwards in the right line in which it was first moved: that line is called the *line of direction* of its motion, and it will not change this direction unless acted upon by another force.

We have hitherto considered forces acting only in the same straight line; but this is not always the case, for it frequently happens that the new force applied makes an angle with the former force, as in the annexed figure. Suppose a body to be moving along the line *ACB*,

Fig. 1.



and when it has arrived at *C*, suppose another force applied in the direction *ED*; there will necessarily be an alteration of the direction in which the body will move, that is towards *ED*. As it cannot move along *AB*, on account of the force *ED*; nor along *ED*, on account of the force *AB*; it follows that it will move in some new direction between them, as *CH*: and it will be shewn in the following chapter, that, in proportion as the new force is greater or less than the former, the line *CH* will be nearer to or farther from the line *ED*.

CHAPTER III.

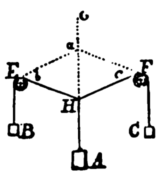
THE SECOND LAW OF MOTION.

When any force acts upon a body in motion, the change of motion which it produces is in the direction, and proportional to the magnitude, of the force which acts.

The second law of motion is of great importance, as it relates to compound motion, and the direction of a body acted on by two forces in different but not contrary directions.

If a body be acted upon by two equal weights, or equal forces of any kind in opposite directions, the body will remain at rest; as for instance, the scales of a common balance, each loaded with a weight of one pound, will be at rest or in equilibrium. In this case equilibrium is maintained by equal forces acting in opposite directions, and this leads to the consideration of equilibrium maintained by the application of these forces. The following is an experimental illustration of what is called the *Parallelogram of Forces*, which is a principle of considerable importance in Mechanics, as it enables us to calculate the combined action of moving powers, as well as their relative effect.

Fig. 2.



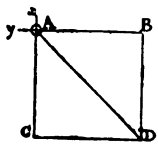
Let two small wheels, *E*, *F*, (fig. 2,)* with grooves in their edges to receive a thread, be attached to an upright board; let the thread be passed over them, having the weights, *B*, *C*, fixed to its extremities. From any part of the thread between the wheels, as at

* In this and the following illustrations no friction is supposed to take place.

H, let such a weight *A* be suspended as will draw down the thread so as to form an angle, *EHF*, and sustain the weights in equilibrium. In this state it is evident that the weight *A*, acting in the direction *HA*, will balance the weights *B* and *C*, acting in the directions *HE*, and *HF*, and these two forces must be equivalent to a force equal to the weight *A*, and acting directly upwards from *H*. To ascertain the relative effect of the weights thus operating, let a line, *HG*, be drawn upon the upright board to which the wheels are attached from the point *H* upwards in the direction of the thread *AH*. Also let lines be drawn upon the board under the threads *HE*, and *HF*; then on the line *HG* mark the point *a*, and let *Ha* represent as many inches as the number of ounces contained in the weight *A*. From the point *a*, on the line *HG*, draw the line *ab* parallel to *HF*, and *ac* parallel to *HE*. Then if the sides *Hb*, *Hc*, of the parallelogram thus formed be measured, it will be found that *Hb* will consist of as many inches as there are ounces in the weight *B*, and *Hc*, of as many inches as there are ounces in the weight *C*. In this illustration *ounces* and *inches* have been used as the subdivisions of weight and length, but it is obvious that the relative weights and lengths might consist of any other denomination of weight and measure; but in every case the same denomination of measure must be applied to all the *lines*, and the same denomination of weight to all the *weights*.

In the preceding diagram (fig. 2) the *Parallelogram of Forces* is represented by the lines *ab*, *bH*, *Hc*, and *ca*, and the line *Ha* joining the opposite angles, which is called the *diagonal*. The sides of the parallelogram, *ab* and *ac*, represent the quantity and direction of the two forces acting together; and the diagonal *Ha* represents the equivalent force, and the object of that illustration was to shew the effect of opposing forces in producing equilibrium; we shall now consider the operation of forces applied in different directions, when their effect produces motion instead of equilibrium. The annexed figure is an illustration of motion produced by two forces in different directions acting on a body. Let a force be applied to the body *A*, fig. 3, sufficient to impel it as far as *B* in a given time, and at the same instant let another force be applied in the direction *AC*, sufficient to carry the body to *C* in the same time; then the body *A* will be under the influence of a compound instead of a single or simple force, and will neither follow the direction of the one force nor the other: for instead of moving towards *B* or *C*, it will move in the direction of the diagonal line *AD*, the length and situation of which may be determined by completing the square *ABDC*, and drawing the diagonal *AD* from the angle *A* to the opposite one. In this case the ball *A* will pass along the diagonal in exactly the same space of time that it would have required to traverse either of the sides of the square, had but one force been applied; thus the ball *A* would reach *D* in the same time that the force *x* would have sent it to *C*, or the force *y* to *B*. It is evident that the force which acts in the direction *AC* can neither accelerate nor retard the approach of the body to the line *BD*, which is parallel to it; hence it will arrive at *D* in the same time that it would have

Fig. 3.

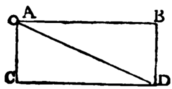


done had no motion been communicated to it in the direction *AC*. In like manner the motion in the direction *AB* can neither make the body approach to nor recede from *CD*: it therefore follows that in consequence of the two motions, the body will be found both in *CD* and *DB*; it will consequently be in *D*, the point of intersection. A ship moving south by a direct wind, may at the same time be carried east just as fast by a tide or current moving east; every instant, therefore, it will go a little south and a little east, and really will describe a middle line pointing south-east. When two forces act upon a body like the wind and tide, the result is the same, whether they act together or one after the other; for instance, if the wind drive a vessel one mile south, as from *A* to *C* (fig. 3), and immediately afterwards the tide drive it one mile east, as from *C* to *D*, the vessel will be in the same place at last, viz. at *D*, as if she had been driven at once south-east in the line *AD* by the simultaneous action of the two. Therefore, by drawing the lines *AC* and *AB* to represent the force and direction of the two causes of motion, and by then adding one of them or an equivalent to the end of the other, as *CD* to *AC*, or *BD* to *AB*, the square or parallelogram is sketched, of which the middle line, or diagonal, as it is called, shews the resultant of the forces, and the true course of the body obeying them.

The single force *AD*, which is thus mechanically equivalent to the two forces *AC* and *AB*, is called their *resultant;* and the two forces, *AC* and *AB*, are called its *components*. When the *resultant* is used for the *component*, the process is called the "composition of force." When we substitute for a single force two or more forces of which it is the resultant, this process is called the "resolution of force."

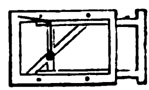
If instead of supposing the two forces equal, the force which impels the body *A* towards *B* is twice or thrice as powerful as that which urges it towards *C*, then the line *AB* must be twice or thrice as long as that from *A* to *C*, and therefore the diagonal will not in this case be that of a square, but of a parallelogram or oblong figure, as in the annexed diagram, fig. 4, and so on for any other proportion of force.

Fig. 4.



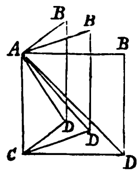
The action of compound forces, and the motion produced by them, may be pleasingly illustrated by a little machine, of which the adjoining cut, fig. 5, is a representation. It consists of two light frames of wood made to slide one over the other; one frame contains a perpendicular wire upon which a ball slides, and a string passes from the ball to the other frame, so that when the frames are moved the motion of the frame constitutes one force, and the pulling of the string the other; and it will be found that, while the ball passes from the bottom or the top of the one frame, it will move over the diagonal of the other.

Fig. 5.



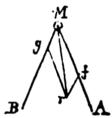
As the diagonal of a parallelogram can never in any case be equal to two of its sides, and the length of the diagonal must diminish as the angles of the sides increase, it follows that resolution of forces must always be attended with loss of power. The annexed diagram, fig. 6, will shew that the greater the angle is at which the forces act, the greater will be the loss by resolution. If *BA*, *AC*, be the sides of a parallelogram, representing the direction of two forces, and *AD* the diagonal line of the body, it is clear that the line *AD* will diminish as the angle *BAC* increases.

Fig. 6.



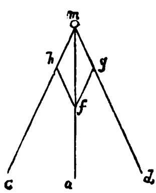
Let *MA*, *MB*, fig. 7, be the directions of two forces acting at the same time upon the ball *M*, let *Mf* represent the *amount* of force which *MA* exerts on the ball, and let the force which *MB* exercises be represented by *Mg*; from *g* draw a line parallel to *Mf*, and from *f* draw a line parallel to *Mg*. A line drawn from *M* to *r* will be the diagonal, or the direction of the resultant.

Fig. 7.



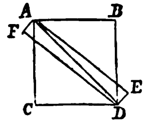
The above illustration shews how two forces may be compounded into one resultant. We will now consider how a single force given as a resultant may be resolved into two forces. Let *ma*, fig. 8, be the *direction* of the force given applied to the ball *m*, and let the *magnitude* of its force be represented by *mf*; let *mc*, *md*, be the two directions in which it is required to resolve it; from *f* draw the lines *fg*, *fh*, parallel to these directions: *mg*, *mh*, represent the amount of the two forces required, *mf* being the resultant, or an equivalent to them.

Fig. 8.



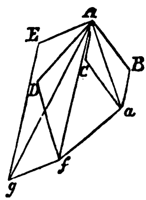
In the annexed figure (9), *AD* is the diagonal of both the square *ABDC* and the parallelogram *AEDF*; from which it may be seen that a body may be made to describe the same diagonal by any two forces represented by the adjacent sides of either a square or a parallelogram.

Fig. 9.



Whatever be the number and direction of the forces which act upon a body at the same time, they may always be compounded into one resultant: for example, if four forces, fig. 10, whose quantities and directions are represented by the lines *AB*, *AC*, *AD*, *AE*, act at the same time upon a body at *A*, first draw *Ba* and *aC* respectively equal and parallel to the lines *AB* and *AC*; we have thus described the parallelogram *ABaC*: if a line be drawn from *A* to *a*, it will consequently be its diagonal, and the direction in which the body would move if only the two forces *AB* and *AC* acted upon it. Having shewn that the body by only one force, represented by the line *Aa*, moves in this direction by the action of the two forces *AB* and *AC*, we will now consider them as only one force, *Aa*, which taken with the force *AD*, gives us the diagonal *Af* of the parallelogram *AafD*, being the direction in which the body acted upon by the three forces, *AB*, *AC*, and *AD*, will move, just in the same manner as if only two forces, as *Aa* and *AD*, had acted upon it. Three forces, therefore, being thus reduced to one, represented by *Af*, we may combine *Af* with *AE*, the fourth and remaining force, and

Fig. 10.



so obtain the diagonal Ag of the parallelogram $AfgE$, the direction in which a body will move by the joint action of four forces. In the same manner may be found the resultant of any number of forces.

Fig. 11.

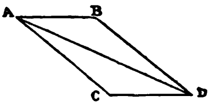
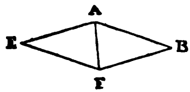


Fig. 12.



If a body be moving in the direction from A to B, fig. 11, with a velocity which would carry it that distance in a minute of time, if it is required to know how much additional velocity it would receive by another force in the direction AC able to carry it from A to C in the same time, we have only to complete the parallelogram and to draw the diagonal AD; and the additional velocity required will be as much as the length of the diagonal AD exceeds the length of the side AB. In like manner, if the force AE, fig. 12, be applied instead of AC, it is clear that this force has a direction contrary to that of the force AB, and therefore will diminish instead of increase its velocity; we must as before draw the diagonal AF, and as much longer the side AB is than the diagonal, so much will be the quantity of velocity lost by means of this new force.

On these principles is established the following fundamental rule: "If a body be subjected at the same time to the action of two moving forces, each of which would separately cause it to describe the side of a square or parallelogram uniformly in a given time, the body will describe the diagonal of the same square or parallelogram uniformly in the same time."

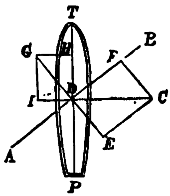
The forces which produce the motions along the sides of the parallelogram are called the *Simple Forces;* and the force which would alone produce the motion along the diagonal is called the *Resulting Force*, and sometimes the *Equivalent Force.*

On the other hand, the force which produces a motion along any line may be conceived as resulting from the combined action of two or more forces. This is exemplified in a boat being drawn along a canal by two horses, one on each side; each pulls the boat directly towards himself in the direction of the rope; the boat cannot go both ways, and its real motion results from this combined action.

This is called the *Composition of Forces.* The two forces having the directions and velocities represented by the force of the horses are said to be *compounded* into one force, which in this case is represented by the onward motion of the boat.

Instances in nature of motion produced by several powers acting at the same time are innumerable. A ship impelled by the wind and tide is an example. When a boat is rowed across a river in which there is no current, it will proceed in a straight line perpendicular to the banks; if there be a current, it will be carried down the river in a direction parallel to the banks; if both forces act upon the boat at the same time, that is to say, if the oars tend to carry it across the river in a direction *perpendicular* to the banks, and the current tends to carry it down the river *parallel* to the banks, it will obey neither of the forces, but will proceed exactly in that intermediate direction which is determined by the composition of force. If it be the object of the rower to gain a point on the other side of the river immediately opposite to him, he will take into consideration the velocity of the current, and will not row in a direct line to the point, but in a slanting direction.

Fig. 13.



Let *TP* be a ship, fig. 13, and *AB* the position of the sail, and suppose the wind to blow in the direction *CD*: if the line *CD* be taken to express the force of the wind, it can be resolved into *ED* perpendicular to the sail, and *FD* in the direction of the plane of the canvass; it is evident that the latter force has no effect in pressing on the sail, and that the former will move the vessel in the direction *DG*. Let *DG* be resolved into *DH* and *DI*, the former *DH* acting in the direction of the keel, and the latter *DI* perpendicular to it, or in the direction of the breadth; *DH* is the only pressure that moves the vessel forward, the other force *DI* urges it sideways. It is evident from the form of the vessel that the velocity in the direction *DH* of its keel is much greater than the sideward direction *DI*. This sideward direction is called lee-way.

It is clear from this explanation that a wind which is nearly opposed to the course of a vessel may, nevertheless, be made to impel it by the effect of sails.

Let *AB*, fig. 14, represent a boat moving in the direction of the arrow *TS*, and let *E*, *F* be two persons sitting opposite each other in the boat: if *E* tosses a ball to *F*, it will appear to move in the line *EF*, whether the boat is at rest or going down the river; if however the boat remains at rest, the ball will really move in that direction, but when it goes along there is another motion (the onward motion of the boat) which acts at the same time upon the ball in the direction *Ea*. The ball is therefore, on the principle of compound motion, carried in the line *Eb*, although it appears to the person throwing the ball to pass in the direction *EF*, because the force which draws the boat carries also himself and the ball with it.

Fig. 14.

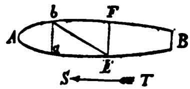
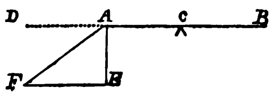


Fig. 15.



Let a uniform rigid bar *AB* balance upon a pivot *C* so as to move freely about it, at the extremity *A* let a force represented in *magnitude* and *direction* by *AF* be applied; then, we may resolve *AF* into two forces acting parallel and perpendicular to *AB*, these will be represented in *magnitude* and *direction* by *AD*, and *AE*. Now *AD* has clearly no effect upon the bar *AB* since it acts along *AB*, and *AB* cannot move off the pivot *C*, therefore the effect of *AF* is just equal to the effect of *AE*, and as the line *AF* is longer than *AE*, its force will therefore be greater: it is proved from this that a force applied at right angles will have an effect equal to that of a greater force applied obliquely. In like manner we may calculate the amount of force which is lost by an oblique application; for suppose in the same figure, the force *AF* is applied, it is obvious that a greater force is employed than is actually necessary, as if it had been applied at right angles, the effect would have been greater: if we would know the exact amount of force employed unnecessarily, we have only to resolve this force into two, viz. AE and EF, and the amount of force that is useless by the oblique application of it will be to the whole as EF is to AF.

Great bells, which are too heavy for one man to ring, are rung by the joint efforts of several men. To the main rope of the bell several other ropes are attached, each being pulled by a man; these various forces may, by the composition and resolution of forces, be compounded into one acting on the main rope.

A paper kite acted upon by the wind and string is another example; so are the motions of fishes, the act of swimming, the flight of birds, &c.

The ease with which equestrian feats are performed, may be accounted for on the principle of compounded motion. When the horseman leaps perpendicularly upwards while the horse is at full speed, on leaving the saddle his body has the same velocity as the horse; he has therefore no occasion to project his body forward, as this is already done for him by the motion he has in common with the horse, which, compounded with the upward motion of the rider, accomplishes the leap: his body therefore, describes the diagonal of a parallelogram, one side of which is in the direction of the horse's motion, and the other perpendicularly upward.

It is important that the student in mechanics should make himself fully acquainted with the principle of the composition and resolution of forces, as it is necessary in the solution of nearly every problem which relates to the motion of bodies acted upon by outward forces, however they may be applied. The doctrine of projectiles, which comprehends the art of gunnery, is founded upon this principle; as it may be demonstrated that all bodies projected by any force in a rectilinear direction near the surface of the earth, will by the force of gravity continually acting upon them, be altered from that direction, and move in a curve towards the earth. By the aid of this principle Sir Isaac Newton has demonstrated in his *Principia* the true system of the universe, and discovered the laws by which the heavenly bodies are directed and regulated, and the effects and influence they have upon each other. It is also of great service in demonstrating the nature and properties of the mechanical powers, of which we shall treat in another part of this volume.

In the preceding remarks on the composition and resolution of forces, we have considered the different forces to act upon the same body so as to move it *uniformly*, and in all such cases it was shewn that the diagonal described would be a straight line; we shall presently consider one of the forces to act in such a manner as to cause the body to move faster and faster, and in this case the line described will be a curve. All bodies which are projected obliquely in a rectilinear direction, and at the same time acted upon by the force of gravity (which has a constant tendency to accelerate their motion), will move in a curvilinear direction. When therefore we see a body move in a curve of any kind whatever, we conclude it must be acted upon by two powers at least, one putting it in motion, and another drawing it away from the rectilinear course in which it would otherwise have continued to move; and whenever that power which bent the motion of the body from a straight line into a curve ceases to act, the body will again move on in a straight line. A ball on being shot from a cannon would, according to the first law of motion, proceed onwards in a straight line, but owing to the force of gravity, it will move in a curvilinear direction, and "the change is proportional to the impressed force." The laws of projectiles cannot be fully understood until the student has made himself acquainted with the "laws of gravitation," which will be treated of in another chapter.

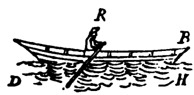
THIRD LAW OF MOTION.

Action must always be equal and contrary to reaction, or the actions of two bodies upon each other must be equal and their directions must be opposite.

When one body strikes against another, the shock is the same, whether the motion be shared between them, or only one of the bodies be in motion: thus, if one man runs against another man who is standing still, each will receive a shock; but if both men be running at the same rate, and in opposite directions, the shock will be doubled. If the weight of one man be much less than that of the other, he will not on that account suffer a greater shock than the heavier man, for the shock which they sustain will be the same, although the former be knocked down and the latter be only made to stagger. When two vessels of 800 tons' burden run foul of each other at sea, their velocities or rates of sailing being equal, each would sustain a shock equal to that which one of them would receive if at rest, and struck by a vessel of 1600 tons' burden moving at the same rate; or if one of the vessels had only 300 tons' burden and the other 800 tons', the shock would still be the same, although the smaller vessel would be much less able to bear it.

If two boats of equal size and weight be floating on the water and at rest, at a distance of four feet from each other, and a man in one boat draw the other towards him by means of a rope, when the boats touch each other each will have been drawn through the space of two feet; or when both boats are together, and the man pushes one boat from him, that and his own boat will recede from each other to equal distances: again, if one boat were twice as large as the other, it would be pulled only half the distance of the small one, thus shewing that *action* and *reaction* in all cases where bodies act upon one another are "equal and contrary."

Fig. 16.



Action and reaction are very plainly seen in rowing, swimming, and flying; as for example, when a man *R*, in the boat *B*, fig. 16, pulls his oar, he drives the water towards *H*, and the water drives the boat as much towards *D*. In swimming, which may be considered rowing with the hands and feet, we are as much pushed forward by the water as we push the water back. Birds in flying are pushed forward by the reaction of the air against their expanded wings when they strike the air with them: for instance, if a bird strikes the air downwards with his wings with a force equal to what would raise a weight of ten pounds, the reaction of the air will push the bird upwards with the same force; but if the bird weighs one pound, the effect of the reaction of the air will cause the bird to rise with the force of only nine pounds,—that is, the bird will rise just as one pound would do fixed to a string passing over a pulley by the force of the descent of ten pounds at the other end of the string. If the bird should strike the air only with a force equal to its own weight, it would then be suspended in the air for some time without motion, which is often observed in kites, hawks, and other birds of prey.

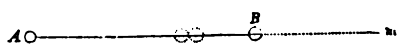
If a piece of wood be pressed by the finger, the finger is equally pressed by the wood. If a ball *A* in motion strike another ball *B* at rest, the motion communicated to the latter will be taken from the former, and the velocity of the former will be proportionally diminished. When this takes place, we say the body *A* has imparted motion to the body *B*, and therefore a part of its own force or motion is destroyed; this is called the *reaction* of the body *B* upon *A*. Now, as the force of the body *A* can be diminished only as far as it finds resistance in *B*, it follows that *action* and *reaction* are equal to each other,—that is, *A* loses as much of its own force as it imparts to *B*. The imparting of motion requires time; this may be shewn by a variety of experiments. One of the simplest is to place a heavy substance, say a piece of copper, on a smooth horizontal surface which covers a vessel of sufficient diameter; when the cover is *suddenly* removed, without changing its horizontal direction, the copper will fall into the vessel, which would not take place if the motion communicated to the cover were at the same time given to the copper.

If a horse draws a heavy load, the load draws the horse equally backwards, for the rope is equally stretched towards both.

In pressing down the empty scale of a balance, while the other scale holds a weight of four pounds, it is obvious that the force exerted must be equal to four pounds; but if one scale be loaded with twenty, and the other with fifteen pounds, the equilibrium may still be preserved by pressing on the latter with a force equal to five pounds only.

Percussion of inelastic bodies.—When two inelastic bodies of *equal moments* moving in opposite directions strike against each other, each will destroy the onward motion of the other, and consequently both will be reduced to a state of rest. If they have *unequal moments* when they come in collision with each other, the motion of the body whose moment was less before the stroke will not only be destroyed, but it will be compelled to move in the opposite direction, as it will follow the impulse of the greater moment; both bodies may then be considered as one mass, moving with a velocity corresponding to the difference between the original movements. Let us suppose the two equal bodies *A* and *B*, fig. 17, are moving against each other, *A* with the velocity 6, and *B* with the velocity 4; then *B* will lose its whole velocity by the stroke, *A* only 4, and the remainder 2 will be divided between *A* and *B*: both bodies will now move with the velocity 1 in the direction from *B* to *m*.

Fig. 17.



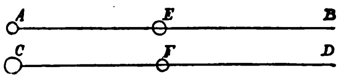
If two bodies, *A* and *B*, fig. 18, be moving in the same direction, *A* with the velocity 6, and *B* with the velocity 4, (the masses being equal,) *A* will overtake *B*, and during the stroke communicate to it as much motion as will equalize their velocities; both bodies will continue to move in the same direction with the velocity 5; *A* will have lost 1, and *B* received an addition of 1 to its velocity.

Fig. 18.

"The velocities with which unequal bodies move after being impelled by equal forces are reciprocally as the quantities of matter in each body;" the meaning of which is, the greater the quantity of matter the less will be the velocity in each. Although the velocities with which *unequal* bodies impelled by *equal* forces are unequal, their moments are however the same: the moment of any body, as before observed, is composed of the velocity with which it moves and its weight multiplied together; therefore, unequal bodies impelled by equal forces, although they will not move with equal velocities, will yet communicate the same force to any body that either of them may happen to strike.

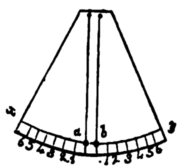
Thus, suppose the body *A*, fig. 19, to weigh one ounce, and let it be impelled along the line *AB*, and suppose another body *C*, weighing four ounces, to be impelled with an *equal force* along the line *CD*, it is evident that their *velocities* will be unequal,—that is, the velocity of *C* will be so much less than that of *A* as its weight is greater than that of *A*; therefore, the weight of *C* being four times greater than that of *A*, its velocity will be four times less. From these remarks it will easily be understood that if the body *A* impinge upon another body *E*, it would impress the *same amount of force* upon *E* as *C* would if it impinged upon a body *F* also at rest; the reason is, that the want of velocity in *C* is compensated by its weight being greater than that of *A*, and the want of weight in *A* is compensated by its velocity being greater than that of *C*.

Fig. 19.



An experimental illustration of the equality of action and reaction in the collision of bodies may be shewn as follows:

Fig. 20.



Let two equal balls, *a* and *b*, fig. 20, formed of clay or any other inelastic substance, be suspended by threads of the same length so as to hang in contact at the middle of the graduated arch *x y*. If they be separated, and the ball *a* be moved to the figure 4 on one side, and the ball *b* to the corresponding figure 4 on the other side, and let fall at the same moment of time, they will impinge upon each other with equal velocities, and they will be found after impact to remain at rest, each having destroyed the force of the other. This proves that when equal masses have equal velocities they have equal forces, for if otherwise, the united masses would after impact move in the direction of that which had the greater force; this may be proved in the following manner.

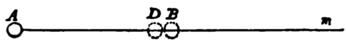
Let the weight of the ball *a* be double that of *b*, let *a* be moved to the figure 3, and *b* to the figure 6 on the opposite side; when allowed to descend, their velocities will be as 3 to 6, and their masses as 2 to 1, and therefore their forces will be equal; because the mass of *a* being 2, when multiplied by its velocity which is 3, the product is 6, and the mass of *b* being 1, and its velocity 6, the product is also 6.

It will be obvious from the preceding examples, that as the *moment* of a body is to be estimated by the velocity of its motion and its weight taken together, a small body may produce an extraordinary effect when moving with great velocity, as well as a very heavy body moving at a slow rate. A tallow candle fired from a gun will pierce a deal board; and a heavily-laden ship with a velocity scarcely perceptible may approach a small boat chained to a pier-wall with sufficient force to crush it. A ball weighing half an ounce shot from a cannon might produce as great an effect as another ball weighing thirty-six pounds, provided the smaller ball had 1152 times the velocity of the larger ball; for 1152 half-ounces being equal to 36 pounds, it is evident that the velocity of the smaller ball would be just so many times greater than the velocity of the larger ball, as the mass of the latter would be greater than that of the former.

Percussion of elastic bodies.—When two bodies, both of them being perfectly elastic, come in collision with each other, the reaction of each of them upon the other must be equal to the loss or gain which it receives from the other: thus, if the one gives the other the impulse

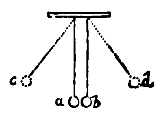
5, it receives by the elasticity of the other the same impulse 5 back again in the opposite direction. If the two equal bodies, *A* and *B*, fig. 21, move against each other, *A* with the velocity 5, and *B* with the velocity 3, then, after impact, *A* will return with the velocity 3, and *B* with the velocity 5. During the stroke *A* lost 3 of its velocity in the direction from *A* to *B* (because 3 is the velocity of *B*), but by the reaction of the elastic body *B*, it receives the whole impulse

Fig. 21.



5 back again in the direction from *D* to *A*, which not only cancels the velocity 2 remaining after the stroke, but impels it backwards with the velocity 3. In the same manner it may be shewn that *B* must return from *A* to *B* with the velocity 5: if *B* stand still, and *A* strike upon it with the velocity 4, then the body *B* will be impelled from *B* to *m* with the velocity 4, which *A* had before the stroke; and as *B* had no velocity, there will consequently be none imparted to *A*, and it will therefore remain stationary at *D*.

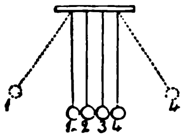
Fig. 22.



Let two ivory balls of equal weight, *a*, *b*, be suspended by threads as in the annexed figure (22), then if the ball *a* be drawn aside to *c*, and suffered to fall against the ball *b*, it will drive it to *d*, or a distance equal to that through which the first ball fell; but it will itself rest at *a*, having imparted its own moving power to the second ball.

If four ivory balls, fig. 23, of equal weight are suspended by threads of the same length, and the first ball be drawn aside and allowed to fall again so as to strike upon the second ball, it will be seen that the second and third balls will remain at rest, while the fourth will bound off with a velocity equal to that with which the first ball struck against the second. In this case the motion, or rather the moving force of the first ball, is transmitted through the two intermediate balls to the fourth, which, finding no resistance, is acted on by the whole force: the same effect would be produced by any number of balls.

Fig. 23.



CHAPTER IV.

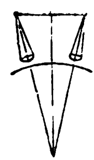
ON GRAVITY.

All bodies when left to themselves fall until they touch the earth, or some other body which can sustain them. This phenomenon takes place at the surface of the earth, at all known heights above, and depths below the surface; as may be seen by hail and rain falling from the clouds, and by a stone suffered to fall into a deep pit. Matter being naturally inert cannot move of itself, and consequently has no power of itself to descend to the earth; it is therefore necessary that some force should cause it to fall, and this force is termed *gravity*. Thus gravity is the force which causes bodies to fall to the earth; but this definition of gravity will convey to us an incomplete idea of its power if we suppose it produces no other effects, for by it are produced many other phenomena and many other motions. As for instance, the flowing of rivers, and the ascent of light bodies in fluids, are but the effects of the same force which we call gravity. Smoke will be seen sometimes to rise to a considerable height in the atmosphere, being driven upwards solely by the force of the medium through which it passes; for the particles of smoke cannot rise in the least degree without displacing or forcing downwards portions of atmosphere equal to their own bulk.

All substances, then, gravitate towards the earth; and this is the cause of what is termed *weight*, being the pressure directed towards the earth which every body exerts upon those which are placed beneath it: if the hand supports a stone, the pressure which the stone exerts upon the hand is called the *weight* of the stone; hence all bodies are heavy, since all fall to the earth when left to themselves.

As the attraction of gravity draws bodies towards the *centre* of the earth, it necessarily follows that two falling bodies will not fall in directions parallel to each other, as two lines cannot be parallel to each other which meet in a point; all bodies therefore, under the influence of gravitation will diverge somewhat from a line perpendicular to an horizontal plane beneath them. If we imagine a pair of scales, as in fig. 24, to be constructed in such a manner as to bear a certain proportion to a sphere towards the centre of which each scale is attracted, and if two lines be drawn from each point at which the scales are suspended to the centre of the sphere, it will be obvious that the scales will diverge a little from the perpendicular. But the magnitudes of any bodies which we can make the subjects of experiment are so extremely inconsiderable when compared with that of the earth as to render their inclination imperceptible to our senses.

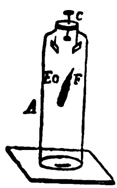
Fig. 24.



Falling bodies.—In observing the fall of heavy and light bodies from the same height we are at once struck with the different rates of their descent: lead falls very rapidly, and paper very slowly; this cannot arise from the difference of the weight of the two substances, as bodies have no natural tendency to fall any more than to rise, and will not fall unless impelled by some force, which force must be proportioned to the quantity of matter it has to move, and as gravity acts equally on all the particles of the bodies, there would be no difference in the time they would reach the earth were it not for some retarding cause. The cause of this difference is the resistance of the air, the effect of which is greater upon the paper than the lead; if however, the paper be rolled up into a ball, it will necessarily offer a much smaller surface to the air, and will meet with much less resistance, consequently it will fall more rapidly.

If therefore, we would ascertain the true motion of falling bodies, it is necessary that they should fall in vacuum, that is, in a space void of air, water, or any other matter capable of offering resistance and hindering the action of gravity. The descent of bodies to the earth when free from the resistance of the air may be pleasingly exemplified in the well-known experiment of the coin and feather. Let *A*, fig. 25, represent a glass receiver having on the top a brass cap or cover fitting it air-tight; through this cover let the wire *C* pass, fitting air-tight also, and supporting a small stage so contrived as to fall when the wire is turned; on this stage place a coin and a feather, *E*, *F*; then exhaust the receiver by means of the air-pump, and turn the wire so as to let the stage drop, and it will be found that they will both fall to the bottom of the receiver at the same instant. This experiment may be modified by allowing a small portion of air to enter the receiver; a slight difference will then be observed in the fall of the two bodies, the feather falling slower than the coin: if more air be introduced, its fall will be still slower, and

Fig. 25.



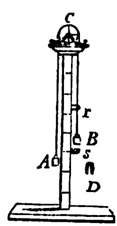
so on; and if the air be allowed to fill the receiver, the fall will be the same as in free air.

We see, therefore, that gravity when acting freely, that is, free from any impediment to its effects, acts upon all bodies with the same energy, whatever be their weight and whatever the substance of which they are composed. In vacuum a mass of gold of one hundred pounds will not fall any quicker than leaf gold or a piece of paper.

After having shewn that in reality all bodies fall with the same volocity, it is necessary to investigate the nature of this common velocity which rules the fall of all kinds of matter. It is evident that if a bullet be dropped from a high tower, having by the force of gravity once acquired a certain degree of motion, it would continue to fall, by the motion it had received by the first impulse, even if the cause were to cease. For instance, if when it had fallen half-way it were possible to deprive it of gravity, it would, according to the first law of motion, continue its motion, and in the direction in which it was first impelled, as a stone continues to proceed when thrown by the hand without any new impulse. The power of gravity, however, does not cease, and therefore every inch the bullet falls it receives an increase of motion. Thus, if in the space of one second it falls through sixteen feet and one inch, it will then have acquired as much swiftness or velocity as will carry it through three times that distance in the next second, five times in the third, seven in the fourth, nine in the fifth. This will account for its accelerated motion, and for the increased momentum with which it falls near the bottom. Thus, the time which a body takes in falling is easily calculated; for if it descends through a certain space* in the first second of its fall, it will descend through four times that space in the first two seconds, nine times that space in the first three seconds, sixteen times that space in the first four seconds, and so on in the same proportion. Thus, to find the space through which a body falls in any given number of seconds, we must multiply the space through which it falls in one second by the square† of the number of seconds in the time of the fall.

Uniformly accelerated motion.—We have shewn above that when a body falls from rest by the action of gravity its velocity goes on continually increasing so long as it falls freely. The motion of bodies which fall freely is so rapid that it cannot be observed with sufficient accuracy, and hence some contrivance is necessary which may diminish the velocity while it preserves the law of the acceleration. This effect may be obtained in different ways, either by making the body descend down a very smooth inclined plane, with an inclination sufficient only to allow its velocity to be observed‡, or by the machine invented by Mr. Attwood. This machine consists of an upright pillar, as represented in the adjoining cut, fig. 26; the weights *A* and *B* are of the same size, and made to balance each other very exactly, and are connected by a silk line that passes over a wheel or pulley *C*. The axis of the pulley is placed upon friction wheels, so that the effect of fric-

Fig. 26.



* It is found by experiment that a body falls very nearly sixteen feet and one-twelfth in the first second by the force of gravity.

† The square of any number is that number multiplied by itself: thus, the square of 2 is 4, the square of 3 is 9, and so on.

‡ This method was invented by Galileo.

tion is scarcely sensible. *r* is a ring through which the weight *B* passes, and *s* is a stage on which the weight rests in its descent. The ring and stage both slide up and down, and may be fixed at pleasure by thumb-screws. The pillar is a graduated scale, and *D* is a small bent bar of metal, and longer than the diameter of the ring *r*. When the machine is to be used, the weight *B* is drawn up to the top of the graduated pillar, and the ring and stage are placed a certain number of inches from each other; the small bar *D* is then placed across the weight *B*, by means of which it is made slowly to descend: when it has descended to the ring, the small weight *D* is taken off by the ring, and thus the weights *A* and *B* are left equal to each other. Now, it must be observed, that the motion and descent of the weight *B* is entirely owing to the gravitating force of the weight *D*, until it arrives at the ring *r*, when the action of gravity is suspended, and the weight *B* continues to move downwards to the stage in consequence of the velocity it had acquired previously to that time. To comprehend the accuracy of this machine, it must be understood that the velocities of gravitating bodies are supposed to be equal, whether they are large or small, this being the case when no calculation is made for the resistance of the air. Consequently the weight *D* placed on the large weight *B* is a representative of all other solid descending bodies. The slowness of its descent, when compared with freely gravitating bodies, is only a convenience by which its motion can be accurately measured, for it is the rate of *increase* of velocity which the machine is designed to ascertain, and not the *actual* velocity of falling bodies.

Now, it will be readily comprehended that in this respect it makes no difference how slowly a body falls, *provided it follows the same laws as other descending bodies;* and it has already been stated that all estimates on this subject are made from the known distance a body descends during the first second of time. It follows, therefore, that if it can be ascertained exactly how much faster a body falls during the third, fourth, or fifth second, than it did during the first second, by knowing how far it fell during the first second we should be able to estimate the distance it would fall during all succeeding seconds. If then, by means of a pendulum beating seconds, the weight *B* should be found to descend a certain number of inches during the first second, and another certain number during the next second, and so on, the ratio of increased descent would be precisely ascertained, and could be easily applied to the falling of other bodies; and this is the use to which this instrument is applied. By this machine it can also be ascertained how much the actual velocity of a falling body depends on the force of gravity, and how much on acquired velocity; for the force of gravity gives motion to the descending weight only until it arrives at the ring, after which the motion is continued by the velocity it had before acquired.

From experiments accurately made with this machine, it has been fully established, that if the time of a falling body be divided into equal parts, say into seconds, the spaces through which it falls in each second, taken separately, will be as the odd numbers 1, 3, 5, 7, 9, and so on. To make this clear, suppose the times occupied by the falling body to be 1, 2, 3, and 4 seconds;

then the spaces fallen through will be as the squares of these seconds or times, viz. 1, 4, 9, and 16, the square of 1 being 1, the square of 2 being 4, the square of 3, 9, and so on. The distance fallen through, therefore, during the second second, may be found by taking 1, the distance corresponding to one second from 4, the distance corresponding to two seconds, and is therefore 3; for the third second take 4 from 9, and therefore the distance will be 5; for the fourth second take 9 from 16, and the distance will be 7; and so on. During the first second then, the body falls a certain distance; during the next second it falls three times that distance; during the third, five times that distance; during the fourth, seven times that distance; and so continually in that proportion*.

Gravitation acts upon all bodies at all times, and that equally whether in motion or at rest; as is evident from the velocities of falling bodies, which are uniformly accelerated during the whole of their course. That a force constantly and equally acting, will produce an uniform acceleration of velocity, is clear from the following considerations.

Suppose a body *A* begins to move by the impulse of gravity impressed at that instant with a velocity 1, the next instant another impulse will create a velocity equal to the former. It will therefore move with the velocity 2, and at the third instant with the velocity 3, &c. for the preceding velocities are not lessened by the succeeding impulses: it therefore follows that if the impulses are equal, and equidistant in time, the motion will be uniformly accelerated, and the velocity will be

* Comstock's Natural Philosophy.

in proportion to the time; so that if a body moves with an uniform velocity for a given time, the space described will be in proportion to that time and velocity taken together. Let one side of the annexed parallelogram, fig. 27, represent the *time* of the motion of a body, and the other the uniform *velocity* with which it moves; the parallelogram itself will represent the *space* described in that time. Thus, let the line *AE* be divided into any number of equal parts, in *B*, *C*, *D*, &c. and from these points draw the *equal* straight lines, *AF*, *BG*, *CH*, &c.; then if *AB*, *BC*, *CD*, &c. represent *equal* successive portions of *time*, and *AF*, *BG*, *CH*, &c. represent the uniform *velocity* with which a body moves, then will the parallelograms *AG*, *BH*, *CK*, &c. represent the *spaces* described in those equal portions of time, and the parallelogram *AFLE* the *whole space* described in the time represented by *AE*. Next suppose that a body moves uniformly as before, during the equal successive portions of time represented by *AB*, *BC*, *CD*, &c. fig. 28, but at the end of each portion of time receives an *increase* of velocity; as, for instance, during the time it moves from *A* to *B* let it move with a velocity represented by *AF*; during the time it moves from *B* to *C* let its velocity be represented by *BH*, &c. If the various parallelograms be completed, then the *space* described in the *time AB* will be represented by

Fig. 27.

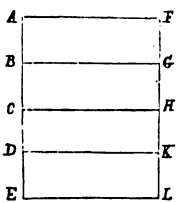
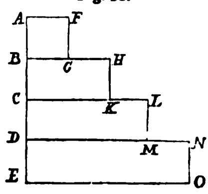


Fig. 28.



the parallelogram *AG*, in the *time BC* by the parallelogram *BK*, &c.

The laws of falling bodies may be rendered interesting by the practical application of them to ascertain the depth of a well: if a stone be dropped into a well, and notice be taken of the time it takes in reaching the bottom, its depth may be calculated on the principles already explained. Suppose the stone is exactly four seconds in reaching the bottom,

then in the first second it will have fallen	16	feet,
in the next second 3 times 16 feet, or	48	,,
in the third second 5 times 16 feet, or	80	,,
and, in the fourth second, 7 times 16 feet, or	112	,,
depth of the well,	256	

The same result will be produced by the following rule, which is more easily remembered, "*The spaces described by a falling body increase as the squares of the times increase.*" Therefore, as the stone takes four seconds to reach the bottom of the well, the square of this is 16; and if this product be multiplied by 16, being the space described by the stone in the first second, we shall have the same result as before, $16 \times 16 = 256$*.

A fragment of rock detached from the summit of a steep mountain begins its motion slowly; but as it proceeds downwards it moves with perpetually increased velocity, gathering fresh speed and momentum with

* Although a body falls nearly sixteen feet and one-twelfth in the first second, we have left out the fraction in the above example to facilitate the calculation.

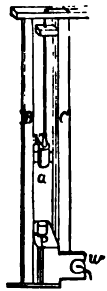
every instant, until its force is such that it sweeps every obstacle before it.

The same principle of accelerated velocity in bodies falling from a height may be illustrated by pouring out molasses or thick syrup: if the height of the fall be considerable, the bulky stream, which on leaving the vessel is perhaps two inches in diameter, is reduced before it reaches the bottom to a small thread; but what it loses in thickness it gains in velocity, as it will fill the receiving vessel with surprising rapidity. A person may leap from a chair without danger; if he leap from a high window, he will probably fracture a bone; but if he jump from the house-top, his velocity becomes so much increased before he reaches the ground as to shatter him to pieces by the fall.

The battering-ram used by the ancients is an example of the accumulation of force, which consisted of a very large piece of timber loaded at one end with brass or iron, and was suspended by ropes or chains from a distance above it, to allow it to swing freely; it was moved by the joint efforts of many men, and when it had acquired by little and little a certain degree of velocity, it was made to strike walls or fortifications of cities, and thus beat them down. This machine was by the men accelerated in an horizontal direction, in the manner that falling bodies are accelerated by gravity in a vertical one.

An engine for driving piles into the earth is an illustration of the accumulation of force downwards, or in a vertical direction.

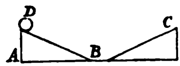
Fig. 29.



It consists of a very heavy piece of hard wood, represented by *a* in fig. 29, usually called the rammer, which is made to slide up and down the upright shafts *BC*. When a pile is required to be driven, the rammer is drawn up to the top of the shafts by means of a rope attached to the windlass *w*, and by an easy contrivance the rammer is loosened from the hook *K* and falls upon the pile *P*. Suppose the rammer weighs 500 pounds, and falls at the rate of 8 feet in a second; therefore, multiplying the mass by the velocity, viz. 500×8 we shall have 4000 for the *momentum* of the rammer with such a fall; and the greater the height from which it falls, the greater of course will be its momentum, or force with which it will strike the pile.

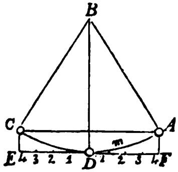
As heavy bodies are uniformly accelerated in their descent, they are as uniformly *retarded* by the force of gravity in their ascent. Thus, if a stone be thrown upwards to the top of a high tower, just as much force must be given it as it would acquire in its descent. The body *D*, fig. 30, in rolling down the inclined plane *A*, will acquire sufficient velocity by the time it arrives at *B* to carry it up nearly to *C*, which is a plane of the same height; and if the plane were perfectly smooth and the air offered no resistance, it would carry it up quite to that point: it is upon this principle the *pendulum* is constructed.

Fig. 30.



A *pendulum* consists of a bob or ball fixed to a small

Fig. 31.



string or wire; if the bob is suffered to fall at *A*, fig. 31, it will fall to *D*, and by the velocity it acquires in the fall will rise to *C*, that is, to the same height it fell from *A* to *D*. This is called an *oscillation;* and if a pendulum were put in motion in a space quite void of air, and free from all resistance from friction on the point of suspension, it would move for ever. The vibrations of the pendulum are produced by its effort to fall; thus, if in the same figure the line *BD* is perpendicular to the horizon, and *EF* parallel to it, then in raising the bob from *D* to *A* it will in reality be raised the perpendicular height *AF*, and it will descend from *A* to *D*, performing the curved diagonal *AmD* with the same velocity as if it had fallen immediately from *A* to *F*. The vibrations of any one pendulum will be described in equal times, whatever be the extent of the arc through which it moves, provided that arc do not exceed a certain limit. It is this remarkable property of the pendulum that makes it so useful as a measure of time. Owing to the resistance of the air the vibration of a pendulum is gradually weakened, and every succeeding arc passed through will be less than the foregoing; and yet it will be found that, although the vibrations of a pendulum become continually slower, there will be but little difference in the time taken up by the bob in moving from 4 to 4, 3 to 3, &c. until its vibration altogether ceases. This equality of vibration of bodies in certain curves was discovered by Galileo, whose attention is said to have been excited by remarking the motion of a chandelier hanging from the ceiling of a church at Pisa; for, observing that it moved uniformly as to time, independent of the space passed through, he was induced to make experiments, which established what has been termed the law of *Isochronism*, or equality of time.

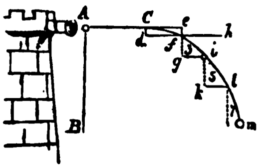
CHAPTER V.

CURVILINEAR MOTION.

Having considered in previous chapters the motion of a body when impelled by a single force—when under the influence of compound forces—and when acted upon only by the force of gravity, the student will now be prepared to comprehend the nature of what is called *curvilinear motion.* This kind of motion, as its name implies, is neither straight forward nor diagonal, but through a line which is curved, and it is caused by the attraction of gravity acting across any body in motion: a stream of water, for instance, issuing from a hole in a cask, as it falls towards the ground is an example of a curved line; and the shape of the curve will depend on the velocity with which the water issues from the cask.

The flying cannon-ball or stone drawn down by the force of gravity is an example, for the projectile force ceases with the first impulse; but the bending force is acting every instant, and by every instant producing a new effect, causes a curvilinear path: thus, in throwing a stone or shooting a ball from a gun, the power of the arm or of the gunpowder will be continually diminishing from the resistance of the air, while the power of gravitation would remain equal were not its effects increased by the accelerated descent of the body in the manner which has been already described in the chapter on gravity; and the consequence is, that these two powers will in all cases so combine into one as to cause the projected body to describe a curvilinear path.

Fig. 32.



In the annexed figure, let *A* represent a ball just discharged from a gun, then *AB* will represent the direction in which the force of gravitation will draw it downwards, and *AC* the direction in which the gunpowder impels it forwards, constituting two forces acting in opposition to each other; but from *A* to *C* it may be presumed that the force of the gunpowder is so much greater than that of gravitation that the latter will not be felt, consequently the ball will proceed in a straight direction for a considerable distance. But the force of gravitation being constant, it may be presumed that at *C* the force of the gunpowder is so far spent as to permit it to begin its descent, and in doing this, if it falls through any given space *Cd*, in passing on from *C* to *e*, it will perform the diagonal *Cf*, and in the next equal space of time will descend through three times the distance *Cd*, or from *f* to *g*, while the force of the powder will be so much more diminished as only to carry it as far as *h*, consequently it will be found at *i*; while in the next equal period it will descend through five equal spaces, as at *k*, while it will only be projected forward to *l*; and in the next period, as it must descend through seven such spaces, it will touch the ground at *m*, and stop, having described a portion of a curve from the point *C* to *m*, or during all the time that the two forces were permitted to act upon it at once.

We have before observed, that the force of gravity being always the same, the shape of the curve described must depend on the force with which the body is projected; but, however great this force may be, the moving body,—a cannon-ball for instance,—if projected horizontally, will reach the ground in the same time as it would if merely allowed to fall by the force of gravity only from the same height: although this fact without consideration appears improbable, it will be easily comprehended, if we bear in mind that the projectile force does not in the least interfere with the force of gravity. If a ball be shot from a cannon with a force sufficient to impel it horizontally with a velocity of 1000 feet in a second, it is nevertheless attracted downwards by the force of gravity with the same amount of force as one with a velocity of only 100 feet in a second; it must therefore descend the same distance in the same time. The *distance* to which two balls will reach depends on the projectile force given to each: if one has more force given to it than the other, it will move through a greater space than the one impelled by the lesser force; but they will both reach the ground at the same time, one moving slowly through a short space, and the other moving rapidly through a greater space.

The curve which a *projectile* describes is termed a *parabola*, although the resistance of the air, which is not recognised in the theory, produces a considerable influence on the practical result.

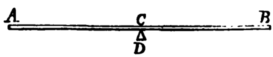
CHAPTER VI.

CENTRE OF GRAVITY.

THE Centre of Gravity is that point of a body in which the whole force of its gravity or weight is united. Whatever therefore supports that point, bears the weight of the whole body; and while it is supported the body cannot fall, because the weights of all its parts are in perfect equilibrium about that point. Thus, if I endeavour to balance a stick by laying it across my finger, after a few trials I find a place where neither end will preponderate; the part, then, which rests upon my finger is immediately under the centre of gravity.

Whenever bodies fall by the force of gravity alone, they fall in the direction of a right line, which may be imagined to be drawn from the centre of gravity of any body towards the centre of the earth, and on this account is called the *line of direction*.

Fig. 33.



If a straight rod of wood or metal, *AB*, fig. 33, of the same thickness and density from one end to the other, be supported by its middle, like a weighing beam, upon the top of the pin or point *D* immediately under its centre *C*, the two ends will just balance each other, and the beam will be supported without any other assistance; this is in accordance with the law of gravity already explained, for as there is an equal quantity of matter in each end, or between *A* and *C* and *B* and *C*, there will also be just as much attraction, and therefore a balance must ensue. To render this more intelligible, let us suppose *A* and *B*, fig. 34, to be two equal particles of matter connected by the straight bar *AB*, a point midway between *A* and *B* will be the centre of gravity of the two bodies; for it is clear that, if *G* be supported, the two bodies will balance themselves about it. The *pressure* upon *G* will be equal to the *weight* of the bodies *A* and *B*; it will therefore be the same whether the bodies be placed at *A* and *B*, or *one* body equal in weight to *A* and *B* together be placed at *G*. The same may be said with respect to the bodies *A*, *B*, *C*, *D*, &c. fig. 35, which are disposed *uniformly* along the inflexible rod *AN*, viz. that the pressure of *A* and *N* is the same as if both were placed at *s*; of *B* and *M* the same as if both were placed at *s*; and so on with all the rest, so that the *whole pressure* of the particles *A*, *B*, *C*, *D*, &c. is the same as if $A + B + C +$ &c. $+ F + K +$ &c. were placed at *s*.

Fig. 34.

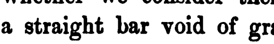
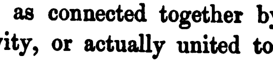


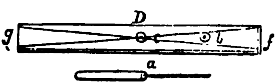
Fig. 35.



It is evident that the number of particles *A*, *B*, *C*, *D*, &c. might be increased till they became contiguous to each other; and the effect is the same whether we consider them as connected together by a straight bar void of gravity, or actually united together by the *power of cohesion.*

In the two preceding illustrations the centre of gravity is also the centre of magnitude or dimensions; but this only occurs when the body possesses equal density in all its parts: if, for instance, the bar *D*, fig. 36, instead of being of uniform density throughout, the end from *f* to *c*

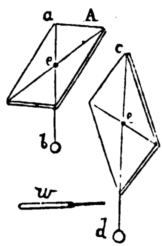
Fig. 36.



be plugged with lead, or any heavy material, and that from *c* to *g* is of wood, then *fc* will contain a greater weight of matter than *gc*, and will consequently preponderate if the pin *a* be passed through the centre at *c*; but if it be removed from *c* to a point *b*, which will now be its centre of gravity, it will balance, though it will now be seen that the centre of gravity no longer coincides with the real centre or middle of the bar *D*.

The difficulty of balancing or supporting tall bodies arises from the circumstance of the centre of gravity always endeavouring to get under the point of suspension. In a suspended body, as the lowest situation which the centre of gravity can find is when it is immediately under the point of suspension, every body hanging freely must have its centre of gravity directly under this point.

Fig. 37.



The following is a simple practical mode of finding the centre of gravity of irregular masses, but with plane surfaces. Thus, suppose *A*, fig. 37, to be a piece of plank, and let a hole be made at any of its corners, as *a*, large enough to introduce a wire *w*, which will support it, and upon which it can move freely; then the wire will be the point of suspension, and a plumb-line with a weight hung upon the same wire, as at *ab*, will represent the line of direction, and the centre of gravity of the plank must be somewhere in the direction of the plummet. The line of direction which the plummet takes on the piece of plank having been marked, then make another similar hole in some other corner of the body, as at *c*, and introduce the wire and plumb-line into this as before; the line of direction now shewn will be *cd*, and in some part of this the centre of gravity must also be; but as it cannot be in two places, the point *e*, where the two lines cut across each other, will indicate the centre of gravity.

As a body of any kind cannot retain its position unless its centre of gravity be supported, it follows that stability will be preserved, if a line drawn from that centre vertically towards the earth falls within the base of the body in question: this may easily be shewn when the bodies are portable; but when they are fixed they will not admit of this kind of calculation, and in this case their centre of gravity can only be ascertained by experiment or calculation*, in which the weight, density, and situation of the respective materials must be taken into account, and having so ascertained the place of the centre of gravity, it may then be seen whether such a body be firmly supported.

The same kind of calculation applies equally to leaning towers and steeples, of which there are many examples in the world. The tower of Pisa in Italy is one of the most remarkable among these, it being 182 feet high, and leans no less than 16 feet out of the perpendicular; and has done so for centuries, and will probably endure centuries longer. The two steeples at Bologna also lean, and were described as doing so before the year 1580. At Caerphilly Castle, near Llandaff in South Wales, the south-east tower, which is hardly 80 feet in height, is 11 feet out of the perpendicular. There are similar examples at Corfe Castle in Dorsetshire, at Bridgenorth, and several other places.

* The mathematical student will find this subject treated of at large in "*Gregory's* Mechanics," 3 vols.

The tendency of a body to fall when the line of direction falls without its base will be readily seen from the following illustrations.

If a body (fig. 38) be placed at the edge of a table, and on a plumb-line being suspended from its centre of gravity the line of direction falls within the base, the body will stand, because the centre of gravity is supported; but if it fall without the base, the body will fall, because unsupported. (The latter experiment may be shewn by reversing the position of the wood, as in fig. 39.) If the line of direction fall exactly upon the edge of the base, the body will be in a state in which the slightest force will overthrow it on that side at which the line of direction falls.

Fig. 38.

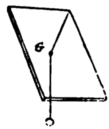
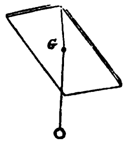
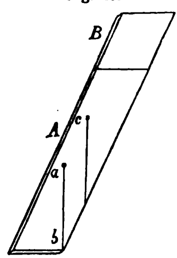


Fig. 39.



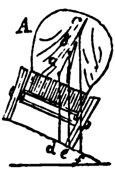
In the annexed figure (40), let *a* be the centre of gravity of the piece of wood *A*, the line of direction *ab* will fall within the base; it will therefore, for the reason above-stated, stand. But if upon *A* another piece of wood *B* be placed, the centre of gravity of the whole will now be raised to *c*; from which point if the plumb-line be suspended, it will be found that the line of direction falls without the base, and therefore the wood falls. For the same reason, when a boat is in any risk of being upset, it is dangerous for the passengers to rise suddenly, as by so doing they raise the centre of gravity, and thus increase the probability of throwing it out of the line of direction.

Fig. 40.



In loading a cart or waggon the heaviest goods should be placed at the bottom, and the lightest at the top; for the lower the centre of gravity is in the cart and its load, the less will be the risk of its upsetting.

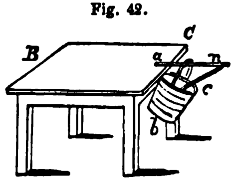
Fig. 41.



Let *A*, fig. 41, be a heavily-laden waggon moving along an inclined road; if the heaviest goods are packed at the bottom, and the lighter goods at the top, the centre of gravity would be in a low position. Suppose this position to be *a*, as represented in the figure; the line of direction *ad* falls within the base, that is, between the wheels of the waggon, which is consequently supported. If the arrangement of the goods be such as to raise the centre of gravity to *b*, the line of direction *be* falls just within the wheel of the waggon, in this case it is liable to be overturned by the slightest jerk; and if the centre of gravity be still higher, as at *c*, so that the line of direction *cf* falls without the wheel, the waggon will be overturned by its own weight.

From what has been stated it is evident that the stability of a body must be increased by lowering its centre of gravity. The manner in which this may be practically illustrated will be easily comprehended from the annexed figure.

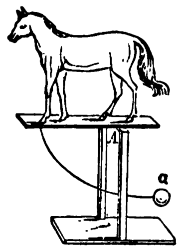
Fig. 42.



From a stick *a*, fig. 42, which of itself would fall, because its centre of gravity hangs over the table *BC*, suspend a bucket *b*, and fix another stick *c*, one end in a notch at *n*, and the other against the inside of the pail at the bottom. Then the bucket will be in equilibrium, as represented in the figure; for the bucket being pushed a little out of the vertical by the stick *c*, the centre of gravity of the whole is brought under the table, and is consequently supported by it.

The effect of placing the centre of gravity of a body in a very low position is shewn in vibrating figures, such as that represented in the following cut. For, if the ball *a*, fig. 43, be removed, the horse will immediately tumble, because unsupported, the centre of gravity being in front of the prop *A*; but on the ball being replaced, the centre of gravity immediately changes its position, and is brought under the prop, and the horse is again in equilibrium. When a man stands upright, the centre of gravity of his body is supported by his feet; if his feet be tied together, and his arms tied to his sides, a slight inclination of the body will carry his centre of gravity out of the perpendicular and he will fall: on extending his legs he stands firm, because his body is supported on a wider base. A man in carrying a burden on his back leans forwards, so as to bring the centre of gravity (of his body and the load) within the basis of his feet. If he carries the load on his head, he will walk erect; and when carrying it in his arms he leans backwards. For the same reason, when we ascend a hill we lean forward, and on descending we lean backward. A large table cannot stand firm on a single leg, unless the leg ter-

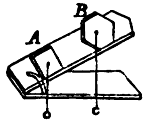
Fig. 43.



minates in a tripod. When a man walks, he throws his body a little forward in order to make the centre of gravity fall in the direction of his toes, and assist by that means the muscular action which propels the body in the same direction. A quadruped never raises two feet on the same side, because the centre of gravity would then be unsupported. A body is stable or firm in proportion to the breadth of its base; hence the difficulty of sustaining a tall body like a stick or a spinning-top upon its point: but it is difficult to upset a cone, because the line of direction falls within the middle of the base, the centre of gravity being low. Rope-dancers, by means of a long pole loaded by weights at the end, perform their feats of agility by dexterously altering the centre of gravity upon each new position which the body takes, so as to keep the line of direction within the base; they fix their eyes on some spot near the rope, and can immediately perceive when it is necessary to alter their position. As the centre of gravity is that point about which all the parts of a body exactly balance each other, it is sometimes so situated as not to be within the body, but in empty space. Thus, a hollow cone, as a common extinguisher, or any body of similar shape, would obviously have its centre of gravity in the space within it. Likewise a piece of wire twisted into the form of a horseshoe, or of a ring, would have its centre of gravity not in the wire, but in the open space within it.

We have shewn that the broader the base is of any body, and the nearer the line of direction is to the middle of it, the firmer it will stand. It is for this reason that a ball so easily rolls along a plane surface; as in all spherical bodies the base is only a point, the smallest point will therefore be sufficient to move the line of direction out of it. It is clear from this that a body *A*, fig. 44, whose line of direction falls within the base, will slide down the inclined plane; but the body *B*, where the line of direction falls without the base, will roll down.

Fig. 44.



Many mechanical deceptions are practised by removing the centre of gravity from its natural into an artificial situation; thus, a cylinder *C*, fig. 45, placed upon a slope or inclined plane *AB*, will naturally descend, because its centre of gravity is thereby approaching the earth; but if a plug of lead be introduced at one side near the edge, as at *e*, which must rise before the roller can descend, the rise being contrary to gravity, its motion down the plane will be arrested; but if the plug of lead be in the position *d*, then the cylinder would fall down to the position *e*, and thus cause it to roll up hill by the action of its weight. The same principle may be illustrated by the following amusing experiment: place a piece of wood turned in the shape of a double cone, fig. 46, united at the base, upon two rulers joined at one end, *A*, and opening a little way at the other, *B*, and raise the open end so as to form an inclined plane; then place the piece of wood at the bottom of the inclined plane and it will roll along to the raised end, appearing to ascend in the direction from *A* to *B*. This is, however,

Fig. 45.

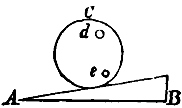


Fig. 46.

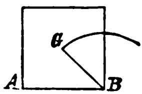
merely an optical deception, for the centre of the double cone, which is its centre of gravity, really descends in obedience to gravity.

A spinning-top cannot be made to stand upright on its peg, owing to the practical impossibility of keeping its centre of gravity vertically over the point of the peg: if however, we spin the top, it will be balanced as long as the rotatory motion continues, because the centre of gravity in each revolution of the top assumes a variety of positions, and has an equal tendency to make the top incline in all directions round it; these opposite tendencies following in quick succession counteract each other as effectually as if they acted simultaneously.

If an oval body be placed on a flat level surface and put in motion, it will oscillate somewhat like a pendulum, because, when disturbed from its middle position, its centre of gravity has risen and seeks to return; the same may be said of the half of any solid globe; and such will always come to rest with its plane face turned directly upwards. This principle may be illustrated by an amusing toy consisting of the half of a sphere made of wood, upon which is placed the figure of a man made of pith; but where the feet should be, the figure has a rounded smooth surface plugged with lead so low as always to allow the figure to sustain an erect position, so that whenever it is pushed down it will immediately rise again.

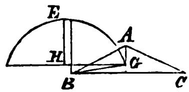
In general the stability of a body depends upon the position of the line of direction, and on the height through which the centre of gravity must be raised before the body can be overthrown. When a body is in the act of falling, its centre of gravity passes through the part of a circle, the centre of which is at the extremity of the base on which the body stands. Let *AB*, fig. 47, represent a square block of wood, the centre of gravity being at *G*. To turn the wood over the edge *B*, the centre of gravity *G* would therefore describe the part of a circle, of which *B* is the centre; and as soon as the centre of gravity *G* passes a perpendicular line over the corner *B*, the wood will fall by its own gravity.

Fig. 47.



This principle will be more easily perceived in the figure of a pyramid. Let *BAC*, fig. 48, be a pyramid, the centre of gravity being at *G*, which is evidently very low, and as the base is broad, a considerable proportion of its whole weight must be raised before the pyramid be overturned; for in order to turn it over the edge *B*, the centre of gravity must be carried over the arch *GE*, and it will therefore be raised through the height *HE*. If however the pyramid were taller and its base narrower, the height *HE* would evidently be less in proportion, and it would be more easily overthrown; the stability therefore of a body, as before observed, depends upon the position of the line of direction, and on the height through which the centre of gravity must be elevated before the body be overthrown.

Fig. 48.



There are certain particular figures, such as squares, parallelograms, and circles, in which when they are of uniform density the place of the centre of gravity can easily be found: as it will be clear from the following illustrations that the materials of the body are similarly distributed around this point, it must therefore be the centre of gravity. Let the annexed figures, 49, 50, and 51, represent thin pieces of card or metal of uniform density, and let them be divided respectively into two equal parts by the straight lines, *A*, *B*, *C*, *D*. Conceive each of these figures to be resolved into *lines of particles* equal and parallel to *AB*, there will then be the same quantity of matter similarly disposed on each side of *AB*; if therefore *AB* be supported, the parts *ACB*, *ADB*, will balance themselves about it; the centre of gravity will consequently be in the line *AB*. For the same reason, because all lines drawn parallel to *AB* are bisected by *CD*, the centre of gravity will also be in the line *CD*; it must, therefore, be in their common point of intersection *G*.

Fig. 49.

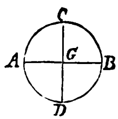


Fig. 50.

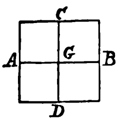
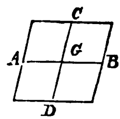
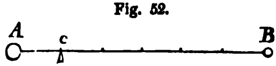


Fig. 51.



The *centre of gravity* is also the *centre of inertia*, for if a bar of wood of uniform density be lifted by its middle, the *inertias* of both ends are equally overcome, and they rise evenly together; but if the bar be lifted up by a part nearer to one end than the other, the shorter end will rise first, because the *centre of inertia* is in the other. The centre of gravity or inertia, however, is not always in the centre of the whole quantity of matter which a body may contain; for if a weight of five pounds *A*, fig. 52, were affixed to one end of a stiff wire or rod, and a weight

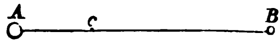
Fig. 52.



of only one pound *B* at the other, they would still be balanced if supported on a point *c* five times nearer to the centre of the large weight than to that of the small one; but although the two bodies *A* and *B* are unequal, yet their masses multiplied into their respective velocities are equal, and consequently a balance will be produced. This fact is, however, explained more at large in the next chapter.

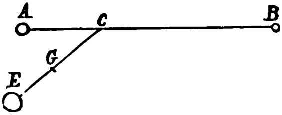
If it is required to find the centre of gravity of two unequal bodies *A* and *B*, fig. 53, we must find a point in the line which joins their centres of gravity, which is distant from the centre of each body in the reciprocal proportion of their masses,—that is, *Ac* must bear the same proportion to *cB* that *B* does to *A*; therefore the product of *A* multiplied by *A c* is the same as the product of *B* multiplied by *Bc*, the *moment* of each being equal; and as the bodies *AB* will balance one another upon *c* if that point be supported, *c* is consequently the centre of gravity of *A* and *B*.

Fig. 53.



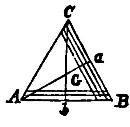
Again, the centre of gravity of three bodies *A*, *B*, and *E*, fig. 54, may be found in like manner; as *c* is the centre of gravity of *A* and *B*, then if *Ec* be joined, and *cE* be divided in *G*, so that *EG* bears the same proportion to *Gc* as the sum of *A* and *B* bears to *E*, the whole will balance on *G*; therefore *G* is the centre of gravity of the three bodies *A*, *B*, and *E*. In this manner the centre of gravity of any system of bodies may be found.

Fig. 54.



The centre of gravity of a triangle may be found in the following manner. Let ABC, fig. 55, be any triangle: draw $A\,a$, Cb, bisecting the opposite sides, and G the point of intersection will be the centre of gravity of the triangle. Let lines be drawn parallel to AB: then it is evident that each of these lines will be bisected by Cb, and therefore the centre of gravity of all of them will lie in Cb. Now the whole triangle may be considered as made up of lines parallel to AB, hence it appears that the centre of gravity of the triangle will be in the line Cb; in exactly the same manner it might be shewn that the centre of gravity will be in the line $A\,a$, and as it cannot be in two places it must be situated at the intersection of Cb and $A\,a$, that is, at the point G.

Fig. 55.



CHAPTER VII.

THE MECHANICAL POWERS.

THE Mechanical Powers are simple machines or instruments by means of which weights may be raised or resistances overcome with the exertion of less power or strength than is required without them.

When forces are so applied to a body as to counteract the effects of each other, the body will be in a state of equilibrium; in this case we have only to consider the relation of the forces which balance each other: this branch of mechanics, as it treats of the action of forces in equilibrium, belongs to the department called *Statics*. When one or more forces act upon a body at rest so as to produce motion, the direction, velocity, and duration of the motion are considered. This branch of mechanics belongs to the department called *Dynamics*.

Machines do not create power, but only convey or modify it; they enable us to apply power in a convenient and economic manner, and in a more advantageous direction than if it were immediately applied to the weight or resistance. If a man could raise to a given height 100 pounds' weight in one minute by the utmost exertion of his strength, no machine could enable him to raise 1000 pounds in the *same space of time;* the mass must be divided into ten parts, and each raised separately: whereas by means of machinery he is enabled to move the whole at *once*, requiring, however, ten times the number of minutes in which he raised the 100 pounds.

Suppose a man has occasion to lift a weight of 40 pounds from the ground and to set it upon a place three feet high; as this weight is easily managed, it might be lifted to the place proposed by his hands. In this case it is clear that the weight will move with a velocity equal to the force, for it is evident that the weight held in his hand will move as fast as his hand does. But if it be required to lift a much heavier weight (say 600 pounds) to the same place, it will be necessary to employ machinery, so that his force may be applied in such a manner that the velocity with which it moves may be as much less than that of the former of 40 pounds, as the weight of 40 pounds is less than 600,—that is, fifteen times less; but the weight being fifteen times heavier, it will, from what we have proved in the foregoing example, move fifteen times slower.

It is necessary to remark, that in the theoretical consideration of machinery the various parts of the machine are considered to be free from friction, and to be absolutely inflexible; they are also supposed to be without weight or inertia. Cords and ropes are considered as perfectly flexible; and when the machine moves, it is supposed to suffer no impediment from the atmosphere. In order to understand the application of a machine, there are four things to be considered: 1. The force or resistance to be sustained or overcome, and which may always be represented by a weight; 2. The force or power which is used to sustain or overcome that resistance, as the force of men or horses, steam, &c.; 3. The *fulcrum*, or prop; and 4. The velocities of the power and resistance. The force or resistance which is to be sustained or overcome is called the *weight;* the force or power overcoming or sustaining this resistance is called the *power*.

The Mechanical Powers are three in number, viz.

1. The Lever.
2. The Pulley or Cord.
3. The Inclined Plane.

These are called by some writers the *Primary Mechanical powers*, and from two of them (the Lever and Inclined Plane) three others are formed:

1. The Wheel and Axle, from the Lever.
2. The Wedge, from the Inclined Plane.
3. The Screw, from the Inclined Plane.

These may be called the *Secondary Mechanical Powers*.

Every species of machinery, however complex, may be resolved into simple elements, which consist only of the above six individual powers.

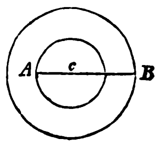
THE LEVER.

The Lever (from a Latin word signifying to lift) is the simplest of all machines, and is only a bar of iron, wood, or other material, supported on and moveable round a prop called the fulcrum.

There are three things to be considered in the lever; the *power*, the *fulcrum*, and the *weight*. The *power* is the force applied, which raises and supports the weight; the *fulcrum* is the prop or support; and the *weight* is the resistance or burden to be raised or sustained. Strictly speaking, the power and the weight are both forces, and they are so named in order to distinguish the one from the other.

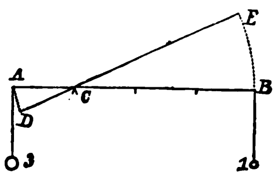
When a lever moves about an axis like a weighing-beam, the different parts have different velocities according to their respective distances from the axis or centre; the truth of this will at once be perceived by the following illustration.

Fig. 56.



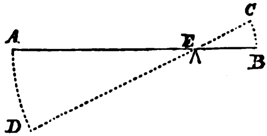
Let *AB* be a straight lever moveable round the fulcrum *c*; the arms *Ac* and *cB* will describe arcs of circles round the point *c*, which will be proportionate to the length of the arms. The velocities also of the arms will be proportionate to the space gone over, that is, to the arc described; therefore in proportion as the arm *cB* is longer than the arm *Ac*, so much will the velocity of the point *B* be greater than that of the point *A*, or in other words the longer the arm the greater will be its velocity.

Fig. 57.



In the annexed figure, 57, let the lever *AB* be supported on the fulcrum *C*, the arm *CB* being three inches in length, and the arm *CA* one inch. Suppose at the end *A* we suspend a weight of three ounces, and at the end *B* another weight of one ounce, the weights and distances will be in reciprocal proportion. In this case then, the lever *ACB* will incline neither way, but remain in a horizontal position; for at the same time that the short arm, being pulled downwards by the weight, describes the arc *AD*, the point *B* being lifted up by the same force will describe the arc *BE*; and as the arcs *AD* and *BE* are proportional to *AC* and *CB*, they will be as 1 to 3: therefore the weight at *A*, which is three ounces, moves over the space *AD*, which is as 1, at the same time that the weight at *B*, which is one ounce, moves over the space *BE*, which is as 3; and as these two tendencies counteract or destroy each other, the two weights tending contrary ways with equal moments, they can neither rise nor fall, but must remain in equilibrium.

Fig. 58.



Suppose the lever *AB*, fig. 58, to be turned on its axis, or fulcrum, so as to come into the situation *DC*; as the end *D* is farthest from the centre of motion, and as it has moved through the arc *AD* in the same time as the end *B* moved through the arc *BC*, it is evident that the velocity of *A* must have been greater than that of *B*. But as the moment is the product of the amount of weight multiplied by the velocity, the greater the velocity the less the amount of weight necessary to get the same product. Therefore as the velocity of *A* is the greatest, it will require less matter to produce an equilibrium than *B*. As the radii of circles are in proportion to their circumferences, they are also proportionate to similar parts of them: therefore as the arcs *AD*, *CB*, are similar, the radius or arm *DE* bears the same proportion to *EC* that the arc *AD* bears to *CB*; and as the arcs *AD* and *CB* represent the velocities of the ends of the lever, being the spaces which they moved over in the same time, the arms *DE* and *EC* may also represent these velocities. It is therefore clear that an equilibrium will take place when the length of the arm *AE* multiplied into the power *A* equals *EB* multiplied into the weight *B*; and consequently that the shorter *EB* is, the greater must be the weight *B*,—that is, the power and the weight must be to each other inversely as the distances from the fulcrum. Thus, suppose *AE*, the distance of the power from the fulcrum, to be 10 inches, and *AB*, the distance of the weight from the fulcrum, to be 5 inches, also the weight to be raised at *B* to be 4 ounces, then the power to be applied at *A* must be 2 ounces, because the distance of the weight from the fulcrum being 5 inches multiplied into the weight, which is 4 ounces, the result is 20 $(5 \times 4 = 20)$; therefore 10, the distance of the power from the fulcrum, must be multiplied by 2 to get an equal product $(10 \times 2 = 20)$, which will produce an equilibrium.

There are three kinds of levers, differing according to the relative positions of the power, the weight, and the fulcrum.

In a lever of the first kind the fulcrum is between the power and the weight.

In a lever of the second kind the weight is between the fulcrum and power.

In a lever of the third kind the power is between the fulcrum and weight.

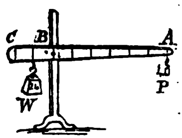
The principal law of the lever is this:—The power is to the weight inversely as the distances from the fulcrum; that is, the smaller the power is which shall be in equilibrium with the weight, the greater must be its distance from the fulcrum. The power is in equilibrium with the weight when its moment is equal to the moment of weight.

First kind of Lever.

The lever of the first kind, where the "fulcrum is between the power and the weight," is principally used for raising heavy weights, or for loosening large stones; and when used in the latter way, it is generally called a crowbar or handspike. Fig. 59 is a representation of the first kind of lever, in which *A* is the end to which the *power* or *force* is applied, *F* is the *prop* or *fulcrum*, and *B* is the *resistance* or *weight* to be raised. According to the rule already explained at p. 53, the longer the acting part of the lever is (represented in the figure from *F to A*), the less will be the power required to move the weight. In the same figure, if *AF* be twice as long as *FC*, then the lever is said to gain a power of two; so that, whatever the weight of *B* is, the power required at *A* to move it will be only half as much as would be necessary to move it without the aid of the lever. If the fulcrum *F* be removed nearer to the weight *B*, so that *FA* is ten times the length of *FC*, then a power of ten would be gained; and so on. In this case, however, *A* would descend ten times as far as *C* would rise, and consequently a very small motion would be given to the weight *B*. *ABC*, fig. 60, is another representation of this kind of lever, in which *B* is the fulcrum. From *A* to *B* is the long arm of the lever, and from *B* to *C* is the short arm. *P* is the power pressing down the long arm at *A*. *W* is the weight suspended from the short arm at *B*.

Fig. 59.

Fig. 60.



The object of this lever is to cause *P*, a small weight, to balance or overcome *W*, a much heavier weight. In all such cases the power will sustain the weight in equilibrium if its *moment* be equal to that of the weight.

The moment of the power is obtained by multiplying the power by its distance from the fulcrum; and the moment of the weight by multiplying the weight by its distance from the fulcrum. Thus, if the number of ounces in *P*, being multiplied by the number of inches in *AB*, be equal to the number of ounces in *W*, multiplied by the number of inches in *BC*, equilibrium will be the result. For example, suppose the weight *P* to be 4 ounces, multiplied by the distance from *A* to *B*, which is 6 inches, the product is 24; and if the weight *W* be 24 ounces, multiplied by the distance from *C* to *B*, which is 1 inch, the product is also 24; the lever is consequently in equilibrium. In like manner, if the weight of 24 pounds be suspended at the extremity of the short arm of the lever, (2 inches from the fulcrum,) and I wish to know what amount of power it is necessary to suspend at the end of the long arm of the lever, I multiply the weight 24 by its distance from the fulcrum, which is 2; its moment therefore will be 48 ($24 \times 2 = 48$). I then divide this product by the number of inches in the long arm, which is 6, and the result is 8, which is the power required ($6 \times 8 = 48$) to be suspended at the end of the long arm in order to balance the weight 24. From the above principles the following rule may be deduced:—Multiply the weight by its distance from the fulcrum, and the power by its distance from the same point, and if the products are equal, the weight and the power will balance each other. It therefore follows that when a small power is required to raise a great weight, it will be necessary either that the power be at a great distance from the fulcrum, or that the weight be brought very near it. The principle in mechanics which produces this phenomenon is called the Law of Virtual Velocities, which is, "That a small weight, descending a long way in any given length of time, is equal in effect to a great weight descending a proportionally shorter way in the same space of time." In other words, what is gained in velocity or time is lost in expenditure of power.

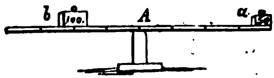
The *mechanical efficacy* of a machine depends on the proportion of the weight to the power, and is said to be greater or less according as this proportion is greater or less. Thus, if in a lever a power of 1 pound support a weight of 15 pounds, the power of the machine or its *mechanical efficacy* is 15. If a power of 2 pounds support a weight of 24 pounds, the power of the machine is 12, 2 being contained in 24 twelve times.

The mechanical efficacy of any lever may be varied at pleasure by changing the positions of the power and the weight with respect to the fulcrum. Whatever be the proportion of the power and weight in any machine, a lever may be assigned in which the power and weight will have the same proportion. Such a lever may be called, in relation to that machine, an *equivalent lever*.

The condition of equilibrium in the straight lever being, that the product of the power multiplied by its distance from the fulcrum should be equal to the product of the weight multiplied by its distance from the fulcrum, it follows that the power may be lessened by increasing its distance from the fulcrum. In like manner, if the distance of the weight from the fulcrum be diminished until the product of the weight multiplied by the distance becomes equal to the power multiplied by its distance from the fulcrum, equilibrium will ensue.

In calculating the proportions between the power and the weight, we must be careful to consider the respective lengths of the long and short arms of the lever. It is indifferent what *units of weight and distance* are taken, provided they be the same at both ends: if inches be the unit of length of the short arm, inches must also be the unit of length of the long arm. In like manner, if ounces be taken as the unit of weight for one arm, it must also be taken for that of the other.

Fig. 61.



The see-saw is an illustration of the first kind of lever. If two boys (one being heavier than the other) place themselves at each extremity of the plank, the *heavier* boy will find himself obliged to move nearer the fulcrum or prop *A*, in order that the plank may be in equilibrium. Suppose the plank to be 8 feet long, and let the weights *a* and *b* represent the boys, the small weight (50 pounds) is 4 feet from the fulcrum, the moment of it is therefore 200 ($50 \times 4 = 200$); and as the large weight is 100 pounds, it must only be 2 feet from the fulcrum to balance the other, because $2 \times 100 = 200$: it therefore follows that, if the arm of the lever which sustains the weight is 2, 3, or 4 times shorter than the other, it is necessary that the weight at the extremity of the long arm be 2, 3, or 4 times lighter, as the case may be, to establish equilibrium.

Many instruments in common use are on the principle of this kind of lever. A long lever turning on a strong iron pin is used by artillerymen in working cannon during battle.

The handspike or crowbar is generally used by masons, builders, &c. for lifting heavy weights through small spaces. A short crowbar is used by thieves in breaking open doors, or wrenching off locks and hinges.

A poker used for raising the coals in a grate is an instance; the bar is the fulcrum, the hand the power, and the coals the weight or resistance to be overcome. In all these cases power is gained in proportion as the distance from the fulcrum to the power or part where the strength is applied, is greater than the distance from the fulcrum to that end under the stone or weight. It is evident that if the long and short arm of the lever be equal, or, in other words, if the fulcrum or prop be exactly midway between the power and weight, no advantage can be gained by it, because they move through equal spaces in the same time: and as it has been shewn, that whenever advantage or power is gained, time must be lost; no time being lost under these circumstances, consequently no power can be gained. A pair of scissors consists of 2 levers of this kind united in one common fulcrum. Thus, the point at which the 2 levers are riveted together is the fulcrum; the fingers are the power, and the substance to be cut affords the resistance; the longer therefore the handles, and the shorter the points of the scissors, the more easily you may cut with them. Snuffers are levers of a similar description, so are most kinds of pincers, the power of which consists in the resisting arm being very short compared with the acting one.

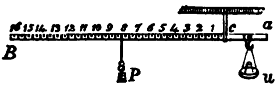
The common balance is a lever; the fulcrum being the point on which the beam of the scales rests, and the weights in the scales are the two forces. This is one of the most interesting and useful applications of the lever.

When equal weights are placed in each scale the beam will maintain its horizontal position, because the distances from the fulcrum when multiplied by the equal weights being in both cases the same, equal products will be the result, and consequently the equal weights will tend equally to move the scales in opposite directions round the pivot, and thus destroy each other's effects, and the whole machine will therefore settle in the same position which it had when the scales were empty. It is evident that the balance will only maintain a horizontal position when the scales have equal weights; for if one weight be greater than the other, the beam will always incline in the direction of the greater weight.

Balances are not unfrequently misconstructed for fraudulent purposes, by making the arm from which the substance to be weighed is suspended longer than that from which the weight hangs, and thus a pound weight will be balanced by a substance in the other scale weighing as much less than a pound as that arm is shorter than the other. The fraud may be detected by transposing the substance to be weighed and the weights. If the object be to ascertain the true weight of the substance, it may be found by the following rule: "Find the weights of the body by both scales, multiply them together, and then find the square root of the product, and this will be the true weight." Thus, suppose a substance weighs 12 pounds in one scale, and in the other only $8\frac{1}{3}$ pounds; 12 multiplied by $8\frac{1}{3}$ gives the product 100, the square root of which is 10, for 10 multiplied into itself gives 100: therefore the true weight of the substance is 10 pounds.

The *steel-yard*, which was in use among the Romans,

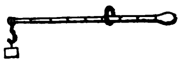
Fig. 62.



and by some writers is called the Roman balance, is another example of this kind of lever. It consists of a lever with a long and short arm: a graduated beam is moveable on the fulcrum or pivot *c*, fig. 62; *a* is the short arm and *B* the long arm; *u* is the scale to receive any article to be weighed; *P* is a small weight moveable on the graduated beam. In proportion as the weight in the scale is heavy, so is the weight *P* moved along to a greater distance from the fulcrum; and when it is moved to a point where it balances the weight in the scale, the figure on the beam denotes the amount of the weight: for instance, if *P* be one pound, and be suspended from the division at 8, it is evident that it will balance a weight in *u* 8 times *P* or 8 pounds.

The Chinese are in the habit of weighing small substances by a light steel-yard; it consists of a wooden rod of about six inches in length, with a silk cord passing through it at a particular part, and knotted below to serve as a fulcrum, and with a sliding weight on the long arm, and a small scale attached to the short one.

Fig. 63.

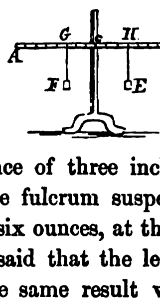


The *Danish balance*, fig. 63, is a steel-yard in which the fulcrum is moveable instead of the weight. Its construction is very simple, being nothing more than a straight bar of iron with a weight fixed at one end, a hook at the other, and a ring moveable along it, which serves as a fulcrum, and from which the whole is suspended. The substance to be weighed is suspended from the hook, and the fulcrum moved to a position which produces equilibrium; the weight is then ascertained by the divisions marked on the bar. This instrument may be graduated experimentally by suspending from the hook successively 1, 2, 3, &c. pounds, and ascertaining the position of the fulcrum which will produce equilibrium.

In making experiments upon the lever, a straight beam of wood divided into equal parts and supported in the middle on a pin, will be found the most convenient apparatus. The following are a few experiments which may be shewn by means of this apparatus; it will be obvious to the learner that they may be altered at pleasure by varying the weights and distances from the fulcrum:

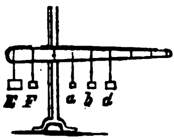
Two equal forces when applied perpendicularly to a straight lever will have the same effect as if the joint amount of the two forces were applied at the middle point between them. Let *AB* be a straight lever, having the arms *Ac*, *Bc*, equidistant from the fulcrum *c*, the lever will therefore be balanced on *c*. Suppose the lever to be divided into inches, suspend a weight *E*, consisting of six ounces, at a distance of three inches from *c*, and on the other side of the fulcrum suspend another weight *F*, consisting also of six ounces, at three inches from *c*; it need scarcely be said that the lever will still be in equilibrium: but the same result will ensue, if, instead of suspending the weight *E* or *F* of six ounces, we suspend *two* weights of three ounces equally distant on each side of either; and it does not alter the effect of these two weights how far from *G* or *H* they be hung, provided they be at *equal* distances.

Fig. 64.



In like manner, if several weights act upon different sides of the fulcrum, equilibrium will be the result, if the sum of the *moments* of all the weights which tend to turn it in one direction be equal to the sum of the *moments* of all the weights tending to turn it in another direction. Thus, in fig. 65, suppose on one side the fulcrum there be suspended three weights; *a* weighing two ounces at a distance of one inch, *b* of three ounces at a distance of two inches, and *d* of four ounces at a distance of three inches; then

Fig. 65.



the moment of *a* is 2×1 or 2
the moment of *b* is 3×2 or 6
and the moment of *d* is 4×3 or 12

the sum of all the moments will therefore be 20

If on the other side of the lever we suspend two other weights; *E* weighing eight ounces at a distance of two inches from the fulcrum, and *F* of four ounces at a distance of one inch; then

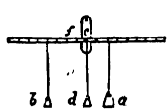
the moment of *E* is 8×2 or 16
the moment of *F* is 4×1 or 4

20

Therefore the *sum* of the moments in each case being the same, the lever will consequently keep at rest.

Suppose a weight *a* of six ounces, fig. 66, be suspended on the lever at a distance of two inches from the fulcrum *c*, then if *two* weights *b*, *d* of three ounces each be suspended at *f*,

Fig. 66.



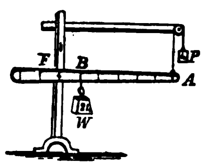
at a distance of *two* inches from the fulcrum on the opposite side, equilibrium will ensue: but suppose each of these two weights to be moved in opposite directions from *f* at the distance of two inches, then the weight *d* will be suspended exactly on the point *c*, and will evidently have no effect in turning the lever either way, and the weight *b* *alone* will balance *a*; because *b* having a weight of three ounces, and being four inches from the fulcrum, its moment is consequently 12 ($3 \times 4 = 12$), which is equal to the moment of *a*, or six ounces multiplied by two inches, its distance from the fulcrum.

It is plainly seen from the foregoing examples that if the number of *ounces* in the weights multiplied by the number of *inches* in the distances be equal on each side of the fulcrum, the lever will be in a state of equilibrium. This leads us to a general property of the straight lever, viz. "Any two weights tending to move a straight lever in different directions, and acting perpendicularly upon the arms, will balance each other, provided the product of the numbers representing the *weights* and *distances* on each side of the fulcrum is the same."

Second kind of Lever.

In a lever of the second kind the weight is between the fulcrum and the power. In the annexed figure the line *A* to *B* represents the long arm, and *B* to *F* the short arm; *W* is the weight to be raised, and *P* is the power. The advantage gained by this lever, as in the first, is as

Fig. 67.



great as the distance of the power from the fulcrum exceeds the distance of the weight from it. Thus, if the power *P* be four ounces, at a distance of six inches from the fulcrum, it will balance a weight *W* of twenty-four ounces at a distance of one inch from the fulcrum, the moments in both cases being the same ($4 \times 6 = 24$, and $24 \times 1 = 24$). If the weight *W* were half-way between *F* and *A*, half of its weight would be supported by the fulcrum *F*, and the remaining half would be balanced by the power at *P*; but in this case the power must be twelve instead of four ounces; and whatever the weight be, if midway between *F* and *A*, it will require but half that weight to balance it at *P*. As the power *P* of four ounces acts over the wheel immediately above it, the axis of the wheel will sustain twice this power, or eight (as we shall have occasion to shew in explaining the nature of the pulley); and if we would know the amount of pressure on the fulcrum *F*, we must subtract the power 4 from the weight 24, which leaves 20; it therefore follows that the total pressure on the fulcrum and the axis of the wheel is 28, which is equivalent to the united forces of the power and weight.

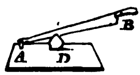
A familiar example of this kind of lever is that of two men carrying a load between them by one or more poles, as a sedan-chair, or as two brewers carrying a cask of beer slung from a pole. In both instances the principle is the same; for either the back or front man may be considered as the fulcrum, and the other as the power. If the barrel be suspended from the centre of the pole, each man will sustain exactly half the weight; but if the barrel be nearer to one man than the other, the man who bears the shorter end of the pole will sustain a greater weight than the man bearing the longer end. Suppose the cask to weigh 3 cwt. and that the point at which it is suspended from the pole is 3 feet from the shoulder of the first, and 6 feet from the shoulder of the second; then the first man will sustain a pressure twice as great as the other man does, for he carries 2 cwt. while the other carries but 1 cwt.; for 2 multiplied by 3 is equal to 1 multiplied by 6.

Another instance of this kind of lever is when a bar is used to raise a heavy bale of goods, by lifting one end of the bar with the hand while the other end rests on the ground. In the annexed figure, 68, the ground is the fulcrum, the force exerted by the hand is the power, and the bale is the weight.

Fig. 68.

The cutting-knife used by druggists and pattenmakers to cut drugs or the wood they use is a lever of the second kind. The joint A, fig. 69, being the fulcrum, the power is applied at the handle B, and the weight or resistance is D, the wood or drug to be cut.

Fig. 69.



A door turning upon its hinges is a lever of this kind. The door is the weight, its hinges the fulcrum, and a person opening or shutting the door by the handle applies the power; and as we have before shewn that the farther the power is from the fulcrum the more easily may the weight be moved, it is evident that there would be considerable difficulty in pushing open a heavy door were the hand applied to the part near the hinges, although it may be opened with the greatest ease in the ordinary way. The consideration of this kind of lever explains why a finger caught near the hinge of a shutting door is so much injured; the momentum of the door acts by a comparatively long lever upon a resistance placed very near the fulcrum.

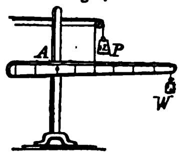
To this kind of lever may be reduced oars, rudders of ships, and common cutting-knives fixed to the working bench. In the case of a man rowing a boat, the water is the fulcrum, the man at the oar is the power, and the boat is the weight. A pair of nut-crackers is an example of the double lever of the second kind; the joint of the crackers is the fulcrum, the nut to be cracked affords the weight or resistance, and the hand which presses the two arms of the crackers together is the power.

Again, two horses may be so yoked to a carriage that each may be made to draw a part proportional to his strength, by dividing the cross-bar in such a manner that the point at which it is fastened to the vehicle may be as much nearer to the stronger horse than to the weaker, as the strength of the former exceeds that of the latter.

The principle of the wheelbarrow may be referred to a lever of this kind. The wheel pressing on the ground may be considered as the fulcrum, the load in the barrow is the weight, and the two handles lifted up by the man represent the power; the nearer the goods in the barrow are placed to the wheel, the more easily may the barrow be lifted, because the power is applied farther from the fulcrum than the weight is.

The third kind of Lever.

Fig. 70.

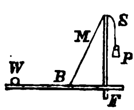


In the lever of the third kind, the power is placed between the fulcrum and the weight. In the annexed figure (70) *A* is the fulcrum, *P* the power, and *W* the weight: in this case the weight being farther from the fulcrum than the power, must pass through more space than it; consequently the power must be greater than the weight, and as much greater as the distance of the weight from the fulcrum exceeds the distance of the power from it,—that is, to balance a weight of four ounces at a distance of six inches from the fulcrum, there will be required a power of twelve ounces if applied at two inches from the fulcrum. Since then a lever of this kind is a disadvantage to the moving power, it is but seldom used, except in cases where velocity and not force is required. This is exemplified in the foot-board of the turning-lathe, fig. 71: the ground on which one end of the plank or board rests is the *fulcrum*, the foot of the workman pressing on the board at a short distance from the fulcrum is the *power*, and the *resistance* is at the further extremity of the plank, which is pulled up by means of the string *A*, attached to a crank above, as fast as it is pressed down by the workman, and thus a constant action is easily produced.

Fig. 71.

As this lever is used only when a great space has to be traversed quickly by the long arm, the power must necessarily always be greater than the weight. When a ladder, being fixed at one end against a wall, is raised into a perpendicular or vertical position by the strength of a man's arm, it is an example of this kind of lever. But the use of levers of the third kind is most beautifully shewn in the animal body, where the Creator has supplied animals with a means to move the limbs with great velocity, by applying the power of the muscles very near the centre of motion, but at the same time giving such a power to the muscles as to enable them to raise the limbs even when great weights are applied at their extremities; as for example, when we lift weights with our hands, or break hard bodies with our teeth: this is of great convenience to the animals, for in almost every case facility and despatch is rather an object than the exertion of great force. To take the arm as an instance; when we lift a weight by the hand, it is effected by means of muscles coming from the shoulder-blade and terminating about one-tenth as far below the elbow as the hand is: now, the elbow being the centre of motion round which the lower parts of the arm turn, the muscles must exert a force ten times as great as the weight that is raised. At first view this may appear a disadvantage, but what is lost in power is gained in velocity, and thus the human figure is better adapted to the various functions it has to perform. In the annexed figure (72) *F* represents the elbow-joint or *fulcrum*; *P* the *power* acting over or from the shoulder *S*, through the contracting muscle *M*, and *B* the arm from the elbow to the hand; and *W* represents the weight placed in the hand. The muscle *M*, on being contracted in a slight degree by the impulse of the

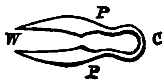
Fig. 72.



will or *power* P, immediately raises the hand in which is placed the weight or resistance towards the shoulder, bending the arm upon the elbow-joint or *fulcrum* F.

The sheep-shears, fig. 73, are two levers of the third kind, the fulcrum for both being at the springing bow at C; the hand applied at PP is the power, and the wool to be cut supplies the resistance.

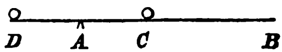
Fig. 73.



The two legs of a pair of tongs are likewise levers of this kind.

In a lever of the second and third kind, if a power support a weight, their distances from the fulcrum will in both cases be in reciprocal proportion. If in the annexed figure (74,) we suppose the line ACB to represent a lever of the second kind, and that a power at B supports a weight at C, then the power and the weight are to each other in reciprocal proportion of their distances; that is, the weight C of 8 ounces, at a distance of 2 inches from the fulcrum A, bears the same proportion to the weight B of 2 ounces at a distance of 8 inches from the fulcrum ($8 \times 2 = 2 \times 8$). Let us imagine the lever prolonged to D, so that AD be equal to AC. It was shewn before that equal velocities caused in equal bodies must be produced by equal powers. But the weight D equal to the weight C, and being at an equal distance from A, will move with the same velocity as C when impelled by the power B; therefore the same power that can support D will support C also. But the line DAB is a lever of the first kind, in which the power B is to the weight D, as AD is to AB. Therefore, since the same power

Fig. 74.



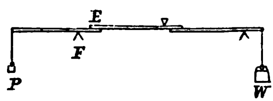
supports *C* equally, it will also be to the weight *C*, as *AD* is to *AB*.

The difference between the second and third kind of levers may be shewn by the following illustration. For if in the same figure we suppose the line *AB* capable by means of a force or forces applied to it to revolve round the point or fulcrum *A*, it is a lever of the second kind, where two contrary forces are applied to it at *C* and *B*, the one at *C* impelling the line downwards, and the other at *B* pushing it upwards; (that is, the *weight* acting at *C*, and the *power* at *B*;) and to make these opposite forces balance each other, the quantity of each force must be in reciprocal proportion to its distance from *A*. But if we suppose the case inverted, and that at *C* the force acts upwards, and at *B* downwards, it is then a lever of the third kind, where the power acts at *C*, and the weight at *B*; but it is obvious that, although the direction of the forces be altered, their quantity must remain the same in order to balance each other,—that is, the power will be to the weight in reciprocal proportion to their distances from the fixed point or fulcrum *A*.

THE COMPOUND LEVER.

A compound lever is formed by connecting several levers together so as to operate one upon the other; they are generally used when great power is required, and it is inconvenient to construct a long lever. The annexed figure (75) represents a compound lever consisting of three levers of the first kind, each working on its own fulcrum. The object of the

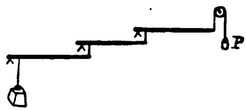
Fig. 75.



machine is to enable a small force at *P* to move or balance a large weight at *W*. In calculating the action of the compound lever, the same rule applies which has already been given for the simple lever, viz.: "Multiply the weight on any lever by its distance from the fulcrum; then multiply the power by its distance from the same point; and if the products are equal, the weight and the power will balance each other." The manner in which the effect of the power is transmitted to the weight may be shewn by considering the effect of each lever successively. The power at *P* produces an upward force at *E*, which bears to *P* the same proportion as *PF* to *EF*. Thus, if we suppose the three levers to be of the same length, the long arms being 8 inches, and the short arms 2 inches, 1 pound at *P* will balance 4 pounds at *E*; because the long arm being 8 inches, being multiplied by the power *P* of 1 pound, gives the product 8, and the short arm being 2 inches, requires to be multiplied by 4 pounds to produce the same result; a power of 4 pounds is thus applied to the long arm of the second lever, which being also 8 inches in length, gives a product of 32, and the short arm being 2 inches, the weight necessary to produce the same result is 16; therefore a power of 16 is applied to the third lever, the long arm of which being likewise 8 inches in length, the result is $16 \times 8 = 128$, and the short arm being two inches in length, must be multiplied by 64 to produce a like result; therefore 64 pounds is the weight which would be supported at *W* by 1 pound at *P*.

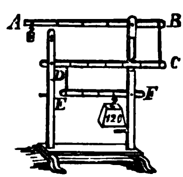
The above explanation of a compound lever, consisting of three levers of the first kind, will be found equally to apply to any system of compound levers. The annexed figure (76) is a system of levers of the second kind; and if the long and short arms of the three levers be the same as in the foregoing illustration, a power of 1 pound will likewise balance a weight of 64 pounds.

Fig. 76.



It will be seen from the above remarks that the effect of any compound system of levers may be found by taking the proportion of the weight to the power in each lever separately, and multiplying these numbers together; and the principles of the calculation will not be altered if they be some of one kind and some of another. The adjoining illustration (fig. 77) represents a compound lever, consisting of one lever of the first kind, and two of the second. *AB* is a lever of the first kind, *CD* and *EF* being both levers of the second kind. We will suppose the lever *AB* to be 5 inches in length, 1 pound therefore at *A* will balance 5 at *B*; as this lever is connected with the second lever *CD*, there will consequently be a power of 5 pounds acting at *C*; and if the length of the lever *CD* be 6 inches, the power of 5 pounds at *C* will sustain a weight of 30 pounds at *D* ($5 \times 6 = 30$). This second lever being in like manner connected with the third lever *EF*, there will be a power of 30 pounds acting at *E*, and the length of this lever being 4 inches, a weight of 120 pounds may be balanced at *F* ($4 \times 30 = 120$). The compound lever is employed in the construction of *weighing-machines*, and particularly in cases where great

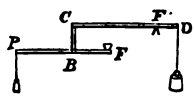
Fig. 77.



weights are to be determined in situations where other machines would be inconvenient, on account of their occupying too much space. The weighing-machine at toll-bars is an example.

Two levers of the first and second kinds are sometimes joined together by a connecting rod *BC*, as in the accompanying figure (78). In this case the resistance *B*, transmitted to *C* by means of the power at *P*, is such that *P* multiplied by its distance from the fulcrum *F*, is equal to the resistance *B* multiplied by the distance *BF*; and the resistance *B*, transmitted to *C*, (which may now be considered as a power,) if multiplied by the distance *CF′*, must be equal to *DF′*, multiplied by *D*, to produce equilibrium.

Fig. 78.



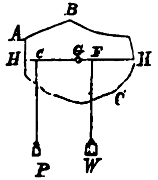
The levers used for raising carriages to take off their wheels are of this class.

ON THE BENT LEVER.

Having considered the various kinds of *straight* levers, we will now take a more general view of this machine, and consider it as any solid body having a fixed axle on which it is capable of turning. This principle being shewn, the bent lever may be more easily understood.

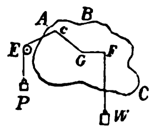
Let *ABC*, fig. 79, be a section of any solid body, moveable on a fixed axis *G*, perpendicular to the paper. Through *G* suppose the horizontal line *HGH* to be drawn, and let the weight *W*, to be sustained, be applied at *F*, and the power *P* which supports it be applied at *C*. The power will sustain

Fig. 79.



the weight in equilibrium, if the number of ounces in the power, multiplied by the number of inches it is distant from the centre *G*, is equal to the number of ounces in the weight multiplied by the number of inches in its distance from the centre. For instance, if the power *P* be 3 ounces at a distance of 6 inches from the axis *G*, it will sustain *W* of 9 ounces at a distance of 2 inches from it: $3 \times 6 = 9 \times 2$.

Fig. 80.

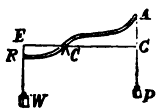


Again, let a weight *W*, fig. 80, be suspended by a cord from the point *F*: this weight will evidently have a tendency to turn the body round in the direction *ABC*. Let another cord be attached to the point *c*, and being carried over a wheel *E*, let a weight *P* sufficiently heavy to turn the body round the axis in the direction of *CBA* balance the opposite tendency of *W*. If the weights *W* and *P* be then ascertained, and also the perpendicular distances *GF* and *Gc* of the cords from the axis be exactly measured, it will be found that if the number of ounces in the weight *P* be multiplied by the number of inches in *Gc*, and also the number of ounces in *W* by the number of inches in *GF*, equal products will be the result.

It is evident that any other denominations of weight and distance besides ounces and inches may be used, provided the *same denominations* be used both with respect to the weight and power. It follows from these examples that the effort of any force to turn a body round an axis, is to be measured by multiplying the force by the perpendicular from the axis on its direction. The product so obtained is called the *moment* of the force round that axis; it is clear that if the moment be increased or decreased in any proportion, the efficacy of the force in turning the machine round the axis is increased or decreased in exactly the same proportion: or if the sum of the moments of the forces which tend to turn the body round in one direction be greater than the sum of the moments of the forces which tend to turn it in the opposite direction, the body will move round its centre in the direction of the former.

The mechanical advantage of the power and the weight of any lever, whether bent or straight, is always represented by a line drawn from the fulcrum at right angles to the direction in which the forces act.

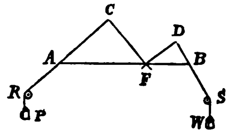
Fig. 81.



Let ACR, fig. 81, represent a bent lever, which balances itself on C, and having two forces P and W applied to it at the points A and R, in the directions AP, RW; if a line be drawn from the fulcrum perpendicular to the direction of the power P, that is, from C to G, and also a line perpendicular to the direction of the weight, which is from C to R, the effect produced may be calculated upon the same principle as that of the straight lever,—for instance, if the distance from C to G be 6 inches, and C to R 3 inches, 2 ounces at P will balance 4 at W $(6 \times 2 = 4 \times 3)$. Any two forces therefore, tending to turn a bent lever in different directions, will balance each other if the *moments* of the two forces be equal.

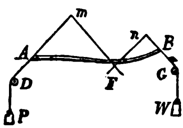
In the foregoing example we have considered the power and weight as acting on the lever, in directions perpendicular to its length, and parallel to each other; but this is not always the case, as both the power and

Fig. 82.



weight often act *obliquely*. Let *AB*, fig. 82, be a lever whose fulcrum is *F*, and let the power act obliquely in the direction *AR*, and the weight in the direction *BS*. In order to calculate the power of this lever, we must draw lines as in the foregoing example; if therefore, the lines *RA*, *SB* be continued, and perpendiculars *FC* and *FD* drawn from the fulcrum to these lines, the moment of the power may be found by multiplying it by the line *FC*, and the moment of the weight by multiplying it by the line *FD*: if *P* be 4 ounces, and *FC* 6 inches, while *W* is 8 ounces, and *FD* 3 inches, equilibrium will be the result, for $4 \times 6 = 8 \times 3$.

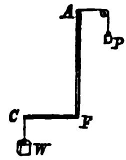
Fig. 83.



The following cut, fig. 83, represents the power and weight acting obliquely, as in the above illustration, but instead of the lever being a straight one, it is bent. The rule of calculation is not affected by the lever being bent; the power therefore of this lever may be ascertained on precisely the same principle as that of fig. 82. Let the line *DA* be continued, and from *F* draw a line perpendicular to it, this line will represent the long arm; in like manner continue the line *GB*, and from *F* draw another line perpendicular to it, this line represents the short arm: the moments of the power and weight may be found as in the former instance.

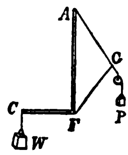
Sometimes the lever is bent in such a manner that the arms are perpendicular to each other; the fulcrum *F*, fig. 84, being at the right angle, such a lever is called a rectangular lever. The weight *W* is suspended from the short arm *FC*, and the power *P* from the longer arm *AF*; the moment of the power in this case is *P* multiplied by *AF*, and that of the weight *W* multiplied by *FC*: if these moments are equal, equilibrium will be the result.

Fig. 84.



If instead of the power acting at a right angle to the fulcrum, we suppose it to act obliquely, as in the annexed figure (85), the method of calculating the power of this lever is the same as in fig. 83. In this case we draw a line from the fulcrum *F* perpendicular to the line of direction of the power *AP*, and this line will therefore represent the true arm of power of the lever: thus, suppose *W* to be 5 pounds, acting at the distance of 1 foot from the fulcrum, it follows that if the line *FG* be 5 feet, and the power *P* 1 pound, the power and weight will just balance each other.

Fig. 85.



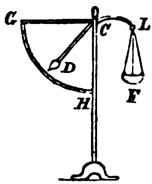
A pump-handle is a familiar instance of the bent lever, in which the force of the man pumping applied at the extremity of the handle is the power; the water to be raised, together with the friction of the piston, is the resistance to be overcome; and the fulcrum is at the joint of the pump-handle.

When the hammer is used for drawing a nail, it is a lever of this kind; the fulcrum of which is the point *C*, fig. 86, on which the hammer presses; the power is applied upon the handle of the hammer at *A*, and the resistance of the nail is the weight.

Fig. 86.

The instrument represented in the annexed figure (87) is called the *bent lever balance*. *L* is the arm of a bent lever, from which is suspended a dish *F* to receive any substance to be weighed; at the extremity of the other arm *CD* a heavy knob is attached, which is moveable upon a graduated arch *GH*. When a weight is placed in the dish *F*, it is evident it will cause the knob *D* to move up the arch; and when *D* has arrived at such a position that it balances the weight, the division to which the index points on the graduated arch expresses the amount of the weight. The positions corresponding to different weights may be determined by experiment or calculation; and these being marked upon the arch, the index at *D* will always point to that division on the arch which represents the weight in the dish *F*.

Fig. 87.

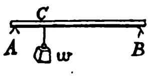


It will be readily seen from the foregoing examples that whatever be the figure or shape of the lever, and whatever the degree of obliquity of the force applied, the power of the machine may be ascertained by drawing ideal lines at right angles from the lines of the forces to the fulcrum, and calculating accordingly.

Having explained the various kinds of levers, we will consider, before concluding this chapter, the effect a weight has upon a beam supported upon *two* fulcrums.

When a beam rests on two fulcrums, *A*, *B*, fig. 88, and a weight *w* is supported at some intermediate place *C*; this weight is distributed between the props in a manner which may be calculated on the principles already explained. If the pressure on the fulcrum *B* be considered as a power sustaining the weight *w*, by means of the lever of the second kind *BA*, then the power multiplied by the length of the lever *BA* will be equal to the weight multiplied by the shorter arm *CA*. If *AC* be one-third, and *BC* two-thirds of the lever *BA*, the pressure on *B* will be one-third of the weight, and the pressure on *A* two-thirds of the weight. It is clear from this example, that if the weight be suspended at an equal distance from *B* and *A*, each fulcrum will sustain half the weight.

Fig. 88.



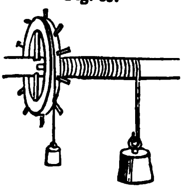
CHAPTER VIII.

THE WHEEL AND AXLE.

It has already been shewn that the lever in communicating motion acts by a succession of short and intermitting efforts. In fig. 58, when the weight has been raised from *B* to *C*, in order to repeat the action, the lever must again return to its first position. During this return, the weight must be supported by some other means. The common lever is therefore only used when a great weight, acting through a small space, is required to be moved by a comparatively small power, and under these circumstances it is well adapted to produce the effect.

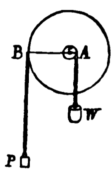
The *wheel and axle* is a contrivance for extending the action of the lever to any distance, and rendering it continuous: it consists of a wheel with an axle fixed to it, so as to turn round with it; the power being applied at the circumference of the wheel, and the weight to be raised is fastened to a rope which coils round the axle. The manner of using this machine is as follows:—The two ends of the axle are supported in an horizontal position, so that the whole machine may freely revolve about the common axis of the wheel and axle: the wheel, by some outward force applied to it, is made to turn round, which causes the axle to turn round with it; this is usually accomplished by means of a rope going round the wheel, and fastened to some place in its circumference. Another rope is also fastened about the axle so as to wind itself round it, while it revolves by means of the force applied to the wheel. The weight is fastened to the end of the rope which hangs down; and as the force acting upon the wheel causes both the wheel and axle to turn round, the rope attached to the axle must necessarily pull up the weight. From this description of the machine, it will be seen that there are two contrary forces acting in opposition to each other; one being the weight tending to make the machine revolve in one way, and the other is the power tending to make it revolve in the opposite direction. These forces act at different distances from the axis; the weight at a distance equal to the radius of the axle, and the power at a distance equal to the radius of the wheel. The annexed figure (89) represents a wheel, and an axle fixed to it, and which moves round with it. If the rope which goes round the wheel is pulled, and the wheel turned once round, it is clear that as much rope will be drawn off as the circumference of the wheel: but while the wheel turns once round, the axle also turns once round; and consequently the rope by which the weight is suspended will wind once round the axle, and the weight will be raised through a space equal to the circumference of the axle. The velocity of the power, therefore, will be to that of the weight, as the circumference of the wheel to that of the axle. If the power bears the same proportion to the weight as the wheel does to the axle, the machine will be in equilibrium; it appears, therefore, that the power of the machine is expressed by the proportion which the diameter of the wheel bears to the diameter of the axle. Thus, suppose the diameter of the wheel to be 12 inches, and the diameter of the axle to be 1 inch, then 1 ounce

Fig. 89.



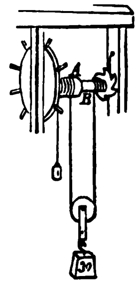
acting on the wheel will balance 12 ounces acting on the axle; and a small additional force will cause the wheel to turn with its axle and raise the weight. The wheel and axle may be considered as a kind of perpetual lever. Fig. 90 represents a section of the machine, and shews how the lever operates. The line going across the machine from *A* to *B* represents the lever, whose centre of motion is *c*: the weight *W*, sustained by the rope *AW*, is applied at the distance *cA*, the radius of the axle; and the power *P*, sustained by the rope *BP*, is applied at the distance *cB*, the radius of the wheel; the long arm of the lever being half the diameter of the wheel, and the short arm half the diameter of the axle: therefore, according to the principle of the lever, we must multiply the weight by its distance from the fulcrum (half the diameter of the axle), then multiply the power by its distance from the same point (half the diameter of the wheel), and if the products be equal, the power will balance the weight. From this it is evident, that the larger the wheel, and the smaller the axle, the stronger is the power of this machine; but the weight must rise slower in proportion. From what has been said, it appears that the *mechanical efficacy* of the wheel and axle may be increased in two ways; either by diminishing the radius of the axle, or by increasing that of the wheel. When, however, this theory is applied to practice, it will be found that, if the weight exceed the power very much, either the axle must be made so slender as not to be able to support the weight, or the radius of the wheel must be so large as to render the machine unwieldy, owing to the power working through an inconveniently great space. The

Fig. 90.



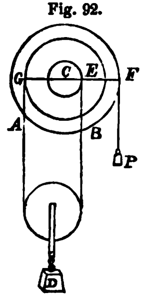
combination of the requisite strength with convenient dimensions and great mechanical power, is accomplished by giving different thicknesses to different parts of the axle. This contrivance is represented in fig. 91. *AB* is the axle consisting of two parts, the diameter of one part being less than that of the other. A rope coiled on the thinner part and passing through a wheel, to which is attached a weight, coils in the *opposite direction* on the thicker part. When the axle is turned in such a direction as to cause the rope to coil round the thicker part, it will necessarily be uncoiled from the thinner part, and upon every revolution of the wheel a portion of the rope, equal to the circumference of the thicker part, will be drawn up, and, at the same time, a portion equal to the circumference of the thinner part will be let down. The result, therefore, of one revolution of the machine will be to shorten that part of the rope where the weight is suspended just as much as the difference between the circumference of the two parts, that is, between the thicker and thinner parts of the axle.

Fig. 91.



The annexed figure (92) represents a section of this machine. It is obvious that the two parts of the rope *A* and *B* equally support the weight *D*, each part being stretched by a force equal to half the weight; and as the machine turns, the rope passes from the small part of the axle to the large part, the power being applied to a rope coiled round the *largest circle*. As the forces at *E* and *F*

Fig. 92.



act on the same side of the centre, it is clear that they will both tend to support the force acting at *G*; and as the pressure of the weight *D* is equally sustained by the two parts of the rope *A* and *B*, the force acting at *E* is equal to that at *G*, and would sustain it without the assistance of *P*, if the distance *CE*, at which it acts, were equal to *CG*. On the principle of the lever, the moments of *P* and *E* must be equal to that of *G*; therefore, if *P* be multiplied by the radius of the wheel, and added to half the weight multiplied by the radius of the thinner part of the axle, we obtain a product equal to half the weight multiplied by the radius of the thicker part of the axle. Hence we perceive, that the power multiplied by the radius of the wheel (which is the lever by which it works) is equal to half the weight multiplied by the difference of the radii of the thicker and thinner parts of the axle.

As *GA* is at a greater distance from the centre than *EB*, it will preponderate unless a force at *F* counteract it: and the more nearly the distance from *C* to *G* and *C* to *E* are equal, the less force at *F* will be required to support the weight. If this system be considered as a lever, the *moment* on the side *CG* is equal to the product of half the weight multiplied by *CG*, and the *moment* on the side *CE* is equal to the product of half the weight multiplied by *CE*, and this moment is opposed to that on the side *CG*. As the moment on the side *CG* is greater than that on the side *CE*, the difference must be counteracted by some power *P* applied at *F*. Suppose *CG* to be 4 inches, *CE* 3 inches, and *CF* 10 inches, the weight of *D* to be 400 pounds, and therefore the weight on each string 200 pounds. On the principle of the lever, the moment of G on the side *CG* is equal to 200 pounds (half the weight of *D*) multiplied by 4 (its distance from *C*) or 800, and the moment of *E* on the side *CE* is equal to 200 × 3, or 600; therefore the *difference* of these moments, to be counteracted by the power at *F*, is 200. To find the amount of weight necessary to be applied at *F*, we must divide the 200 pounds to be supported, by the distance *CF*, which we have supposed to be 10 inches: we therefore obtain a product of 20 (200 divided by 10); 20 pounds at *P* will thus balance 400 pounds at *D* by the aid of this machine.

It often happens that the action of the power is liable to occasional suspension, in which case the weight would rapidly descend, and thus lose the advantage gained by raising it: in order to prevent this, a contrivance called a *ratchet-wheel* is affixed to the axle (see fig. 91). This wheel consists of teeth, all of which are curved in one direction. A bolt or catch working on a pivot falls between the teeth of the wheel. When the axle is turned round, the ratchet-wheel turns round with it, and the catch which falls between the teeth prevents it receding when the action of the power happens to be suspended. By this contrivance the power may be withheld at pleasure, while the effects of its past action may be retained.

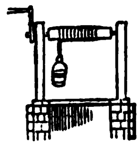
From the above explanation of the wheel and axle, it will be readily seen that the law of *virtual velocities* already explained equally applies to this machine as to the lever; therefore, "If two forces balance each other on the wheel and axle, and the whole be set in motion, the product of the forces multiplied by the spaces through which they respectively move is the same."

Suppose a small weight, fig. 89, suspended to the circumference of the wheel, supports a larger weight suspended to the circumference of the axle, then the small weight will be in the same proportion to the large weight as the radius or circumference of the axle is to the radius or circumference of the wheel: if the wheel and axle be turned once round, the small weight will descend through a space equal to the circumference of the wheel, and the greater weight will be raised through a space equal to the circumference of the axle; and as these spaces are *proportional* to the two weights, the product of the smaller multiplied by the space through which it moves, (that is, the circumference of the wheel,) is equal to the product of the larger multiplied by the space through which it moves (that is, the circumference of the axle).

The same principle applies where one part of the axle is thicker than the other, fig. 91. If the wheel be turned once round, the thicker part of the axle *A*, and the thinner part *B*, will each revolve also; and while one part of the rope is coiled round the thicker end of the axle, the other part of the rope is uncoiled from the thinner; and one end of the rope will therefore be shortened by a quantity equal to the circumference of the thick part of the axle, and the other end will be lengthened by a quantity equal to the circumference of the thinner part of the axle; and the whole rope will be shortened equal to the difference of the circumferences of the two axles; the weight will therefore be raised through a space equal to *half* that difference. Hence we conclude, that the space through which the *power* is moved is to the space through which the *weight* is moved as the radius of the wheel to half the difference of the radii of the two axles, or as *twice* the radius of the wheel is to the difference of the radii of the two axles; therefore the power and weight will balance each other when the power multiplied by the space through which it moves is equal to the weight multiplied by the space through which it moves.

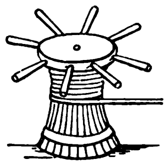
There are various ways in which the power is applied to the wheel. Sometimes, instead of being applied to the axle by means of the wheel, an iron handle is fixed which acts as a lever, and by its circular motion answers the purpose of a wheel, as in fig. 93. By this machine water is raised from a well in a bucket. It often happens that the well is so deep as to cause the rope to coil more than once the length of the axle: in this case it will be found that, as the bucket ascends nearer the top, the difficulty of turning the handle will be increased; for, as the advantage gained is in proportion as the circumference of the wheel is greater than that of the axle, it is evident that when the rope coils round the wheel the *second* time, the difference between the circumference of the wheel and that of the axle will continually diminish; so that the advantage gained is less every time a new coil of rope is wound on the whole length of the axle.

Fig. 93.



Sometimes pegs are placed round the circumference of the wheel at equal distances, to which the hand may be applied as the power (see fig. 89). This manner of applying the power is exemplified in the wheel used to work the rudder of a ship. In the *windlass* the axle is horizontal; but in the *capstan* it is vertical. The advantage of its vertical position in the *capstan* (fig. 94) is very evident. A series of long levers are fixed at equal distances round the axle; to each of these the force of one or more workmen may be applied at the same time by their walking round the axle and pushing the levers before them: then, as these bars are pushed round, the upright axle will evidently turn round with them, and thus the rope is wound round the axle, and the weight is drawn towards it.

Fig. 94.



The capstan is principally used for raising the anchors of ships: when not in use, the spokes are taken out and laid aside.

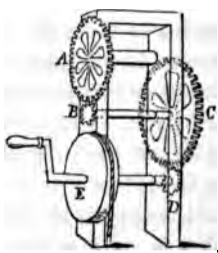
In calculating the power of this machine, the proportions are the same as in the wheel and axle; that as the radius of the wheel (which is in this case half the thickness of the axle added to the length of the bar) is to the radius of the axle, so is a weight supported by a man applying his strength to the end of one of the bars to the quantity of power by which he supports it.

Suppose a heavy stone weighing 20,000 pounds is required to be raised, and suppose 10 capstans to be placed round it, each capstan having 10 bars, to each of which bars a man applies his strength. Let the radius of the axle be 6 inches, and let each bar be 5½ feet in length, and suppose the force of each man equivalent to 200 pounds. If the length of each bar, which is 5½ feet, be added to the radius of the axle, which is 6 inches, the product will be 6 feet, which in this case is the radius of the wheel. Then as 6 inches (the radius of the axle) is to 72 inches (the radius of the wheel),—that is, as 1 to 12,—so is the quantity of each man's force to the weight he is able to support by one bar of this machine; therefore, as 1 to 12, so is 200 pounds to the weight each man sustains,—that is, 2,400 pounds. Now this multiplied by 100 (there being 10 capstans, each consisting of 10 bars), produces 240,000 pounds, which is more than the weight of the stone to be raised. It was chiefly by the help of a series of capstans that Domenico Fontana, an Italian architect, removed the great obelisk in the centre of the portico before St Peter's Church at Rome, the weight of which was nearly a million pounds.

We have already explained, that, if in the wheel and axle the weight exceed the power very much, either the axle will be liable to break, or the wheel will be of an unwieldy size; when therefore great power is required, wheels and axles may be combined in a similar manner to a compound system of levers, and the conditions of equilibrium are exactly the same in both.

In compound wheel work, the power is applied to the circumference of the first wheel, which transmits its effect to the circumference of the first axle. The axle being made of sufficient strength and thickness to support the weight, the wheel belonging to it is made of such a radius as is found most convenient, and the circumference of the wheel is cut into teeth as in the annexed figure (95). This wheel is made to play upon a small wheel which is called the *pinion;* this pinion is also fastened to another axle, and so placed that the teeth of the wheel and *pinion may take* hold of each other

Fig. 95.



as they both turn round. At the end of this second axle is fastened another large wheel, the circumference of which is likewise divided into teeth; and under it is placed a second pinion fastened to another axle. At the other end is fastened another large wheel, to which the force that turns the whole machine is to be applied. All these wheels are represented in the adjoining figure. In this combination of the wheel and pinion, a long perpetual lever works against a short perpetual lever, by which a considerable mechanical advantage is gained. By this combination the same effect is produced as if the radius of the first wheel had been increased; but it is obvious it is done with much more ease and convenience: the power of the wheel and axle under these circumstances may be found by multiplying together the powers of the several wheels of which it is composed. This power is generally computed by numbers expressing the proportions of the circumferences or diameters of the several wheels, to the circumferences or diameters of the several axles respectively. Suppose it were required with a power equal to 40 pounds to sustain a weight of 4,320 pounds, which is to the power as 108 to 1. Suppose also, that the diameter of the axle is 8 inches, and just able to support such a weight without breaking. Now if we make use of the simple machine to sustain this given weight, we shall find that it will require to an axle of 8 inches diameter a wheel of 72 feet, 864 inches in diameter, to sustain the weight*.

* If the weight, which is equivalent to 108, be multiplied by the diameter of the axle, which is 8, the product is 864; in order therefore to balance this, the diameter of the wheel must be 864 inches, as $864 \times$ by the power $1 = 864$.

But such a machine would obviously be unwieldy and inconvenient, therefore the compound wheel must be used.

We have already shewn that if a power be equal to 1 and its velocity 20, and a weight be equal to 20 and its velocity 1, the power and weight will support each other. If therefore the power, by the help of this compound wheel, be so increased as when multiplied into its quantity, it is equal to the velocity of the weight also multiplied into its quantity, the power applied to this machine will sustain the weight. In the same figure let the uppermost wheel be *A*, the pinion under it *B*, the second wheel *C*, the second pinion *D*, and the third wheel *E*. Let the wheel *A* be 12 inches radius (that is, 24 inches in diameter), and the radius of the axle 4 inches; if the power be applied at the circumference of the wheel *A*, its velocity will be to that of the weight as the radius of the wheel to the radius of the axle,—that is, 12 to 4, or 3 to 1. Let us now suppose the radius of the pinion *B* to be 2 inches, and the number of its teeth 12, and the number of teeth in the wheel *A* to be 72; their proportion will therefore be as 1 to 6. Let the wheel *C* be equal in radius and in number of teeth to the wheel *A*. The weight being supposed to act upon the axle of the wheel *A*, makes the circumference of that wheel revolve with a velocity that is to its own as 3 to 1 (the radius of the wheel being 12 inches, and the radius of the axle 4). As its teeth go round, they catch successively upon those of the pinion *B*, and make that pinion go round with a velocity equal to their own,—that is, 3 times faster than the weight. The pinion *B* in turning round makes its axle and the wheel *C* to turn round with it; and as the radius of the wheel *C* is to that of the pinion

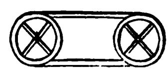
B as 6 to 1, the velocity of its circumference, and therefore of the power supposed to be applied there, will be to that of the pinion *B* as 6 to 1 also. But the pinion *B* goes round 3 times faster than the weight; therefore the velocity of the power at the wheel *C* being 6 times greater than that, must be 18 times greater than that of the weight. Let us now suppose the power applied to the circumference of the wheel *E*, and let the pinion *D* be equal in radius and in number of teeth to the pinion *B*, and let the radius of the wheel *E* be equal to that of the wheels *C* and *A*; as the teeth of the wheel *C*, taking hold on those of the pinion *D*, causes that pinion to turn round with a velocity equal to its own, this pinion makes the wheel *E* turn round also with a velocity that is at the circumference as the radius of the wheel to the radius of the pinion,—that is, as 6 to 1. We have already shewn that the velocity of the pinion *D* is 18 times greater than that of the weight, and as the velocity of the power at the circumference of the wheel *E* is 6 times greater than that, it will therefore be to that of the weight as 6 multiplied by 18 is to 1,—that is, 108 to 1. Hence it follows that the power whose quantity is as 1, has a velocity of 108; and the weight whose quantity is as 108 has a velocity of 1; from which we conclude that the moments of the power and the weight are equal, and will therefore mutually support each other.

When one wheel acts upon another by means of teeth, in order that the motion may be communicated from wheel to wheel with uniformity, the teeth must be so cut that they may turn smoothly upon one another; it is necessary that great attention be paid to the exact adjustment of the teeth, or they will be apt to jar upon and break each other.

In the wheel and pinion it is evident that each pinion revolves much more frequently than the wheel which it drives; and the number of revolutions which a pinion connected with a wheel will make for one revolution of the wheel, may be known by the number of teeth in each. Thus, if the wheel contains 100 teeth, and the pinion only 10 teeth, the pinion will revolve 10 times to 1 revolution of the wheel. Therefore the respective revolutions of every wheel and pinion working together will have the same proportion as their number of teeth taken in a reverse order.

Sometimes motion is transmitted from one wheel to another by means of straps or cords; one great advantage attending this is that the wheels may be placed at any distance from each other, and be made to turn either in the same or contrary directions: when however the belt is very long which connects the two wheels, it is apt to vibrate in its motion, from its weight being unsupported at the centre. The annexed figure (96) represents the transmission of power by a belt from one wheel to another, both being of the same diameter. The power which is given to one wheel is transmitted to the other; and as they are both of the same diameter, their velocities will be the same, and both will turn round in the same direction.

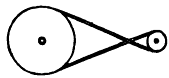
Fig. 96.



If the diameter of one wheel be greater than that of the other, the smaller wheel would turn round more frequently than the large one. For instance, if the diameter of the large wheel were 9 inches, and the diameter of the small wheel 3 inches, the latter would turn round three times for one revolution of the former.

The annexed figure (97) represents two wheels moving in different directions, which arises from the belt being crossed upon leaving the large wheel: besides the advantage of changing the rotatory motion of wheels which the crossing of the belt produces, it has also the advantage of causing the belt to move more steadily.

Fig. 97.



In watch and clock work the wheels are used to produce and regulate motion merely, without any reference to weights to be raised, or resistance to be overcome; the force which is applied varying in its intensity, while the wheels are required to be kept at a uniform speed. Suppose that the power is a spiral spring of tempered steel (fig. 98), the force of which lessens as it relaxes, it is evident that its force is greatest when it *begins* to relax; the following illustration, which represents the apparatus of the watch, will exemplify how this defect is compensated. The spiral spring is coiled up and put into a brass box called the barrel *B* (fig. 99). The spring is fastened to the barrel by an oblong slit at its outer extremity; a central axis or spindle goes through the brass box, to which the other end of the spring is attached. A chain is coiled round the barrel *B*, one

Fig. 98.

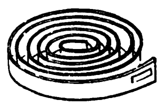
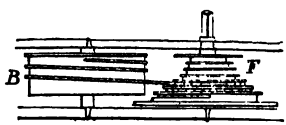
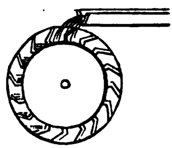


Fig. 99.



end of which is fixed on the barrel, and the other is attached to a brass cone *F*, which is broad at the bottom and narrow at the top, which has also an axle on which it is capable of turning. This cone is called a *fusee*. When the chain is rolled upon the spiral thread of the fusee, the spring within the barrel *B* is relaxed. When, by means of a watch-key, the spindle is made to turn, the chain is drawn from the barrel and wound round the fusee; the chain in rolling off the barrel causes it to revolve, and consequently to wind up the spring inside. When the force of the spring is greatest, the chain turns round the smallest part of the fusee, and may then be considered as acting with a small lever: as the spring gradually relaxes, the chain is drawn *off* the fusee and *on* to the barrel, it then gains a greater lever advantage, as the part on which it acts is nearer to the base of the cone. Thus the gradual loss of force is counterbalanced by a gradual increase of lever advantage; so that, these two opposite effects producing a mutual compensation, an uniform action is the result*.

Fig. 100.

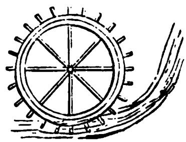


In water-wheels the water is applied by causing it to fall or flow into buckets at the circumference of the wheel, as in fig. 100; in this case the apparatus is called an *overshot wheel:* the water is made to approach

* The wheel-works of the watch are of too complicated a nature to admit of a detailed explanation in this little volume. We have endeavoured above to explain the action of the fusee, as this part of the apparatus is more immediately connected with our present subject.

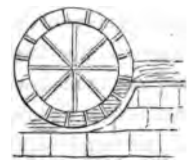
by a channel on a level with the top of the wheel, and falling into the buckets turns the wheel by its own weight. Sometimes the impulse of the water acts against flat boards at the circumference, as in fig. 101; it is then called an *undershot wheel.*

Fig. 101.



Both these principles act in the *breast wheel*, fig. 102, which is used when the water cannot be made to approach the wheel higher than a point opposite the middle of the wheel.

Fig. 102.



The water which is brought by an artificial channel flows into the buckets fixed at the circumference of the wheel. The wheel is turned round by the weight of the water in the buckets; and when these arrive at the bottom in revolving, the water flows from them, and they ascend empty on the other side to be again filled.

Windmills are used only in cases where regularity and constancy of motion are not requisite. The power in this case is the force of the wind acting on various parts of the arms, and may be considered as different powers acting on different wheels having the same axle.

Numerous methods have been contrived in order to combine the weight and muscular power of cattle in giving motion to wheels. One instance is that of a horse placed at the circumference of the wheel, and moving forwards on the side of the wheel on which he steps, and as the wheel descends the horse maintains his position continually.

One of the most common practical applications of the wheel and axle is the crane used on wharfs and in warehouses for raising and lowering heavy parcels of goods. Formerly it was a very common practice to work such cranes by a man walking in the inside of the large wheel, which was thus put into motion in the same way that circular cages are moved by an inclosed squirrel, or that the turnspit dog was formerly employed for roasting meat. This was, however, not only a dangerous but a very disadvantageous mode of employing strength, and is now almost entirely abandoned. The best and most efficacious method of employing human power to a vertical wheel and axle is to cover the exterior circumference of the large wheel with float or projecting boards like a common water-wheel, and to permit a man or men to tread upon the boards as in walking, at the height of a line which forms the horizontal diameter of the wheel, for then the man's whole weight and power will be thrown into action at the point where it will have the greatest effect. This same application of power has been resorted to as a means of employing the criminal prisoners in most of the gaols of Great Britain under the name of the treadmill.

The wheel and axle is sometimes used to multiply motion instead of to gain power, as in the multiplying wheel of the common roasting-jack, to which it is applied when the weight cannot conveniently have a long line of descent: a heavy weight is in this case made to act upon the axle; while the wheel by its greatest circumference winds up a much larger quantity of line than the single descent of the weight would require, and thus the machine is made to go much longer without winding than it otherwise would do.

Among all the simple machines, there is none so generally useful, and therefore so frequently making a part of compound machinery, as the wheel and axle: its advantages are partly owing to its motion being revolving, which is capable of being uninterruptedly continued through a period of indefinite extent; and to this advantage may be added the extreme facility with which wheels may be connected in various modes with other kinds of machinery. Hence there are few complex machines of which wheels do not constitute the most effective or essential parts. Thus are formed a vast variety of mills, from the coffee-mill to the powerful and complicated engine called a rolling-mill for compressing plates of iron and cutting them into rods or bars; all the numerous kinds of wheel carriages, turning lathes, and grinding machines; clocks, watches, and timekeepers in general; spinning jennies, and many other machines used in the cotton, linen, woollen, and silk manufactures; and steam-engines, under many of their modifications, to accommodate them to the purposes to which they are devoted.

CHAPTER IX.

THE PULLEY OR CORD.

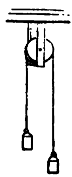
THE machines we have hitherto considered have been supposed to be constructed of *inflexible* materials; but the mechanical efficacy of the machine we are now about to explain depends upon the *flexibility* of the material. A *cord* may be used to transmit a force from one direction to another, and this is one of the greatest conveniences attending this machine: thus, suppose it be required to support a weight *C*, fig. 103, by means of a power in the direction *a*; this may be accomplished by attaching one end of a rope to the weight, and passing it over the edge *a*, the power being applied by the hand. If the edge or prop over which the cord acts be a projecting piece of wood or iron, the exertion of the power would evidently produce considerable friction on the rope and cause it to wear away very quickly: to obviate this defect the pulley was invented, which consists of a round block of wood having a groove cut in its edge of sufficient breadth and depth to admit a rope; this block is made moveable about its centre by means of a pin passing through it, which is supported in a frame called a *sheaf*. The pulley being capable of revolving round the pin, will obviously remove in a great degree the effects of the friction of the rope: it is necessary to observe that although the name *pulley* has been given to the block and sheaf, the mechanical efficacy of this machine does not depend upon these, but upon the cord, as we have already explained, the block

Fig. 103.

and sheaf being only introduced to obviate friction. Pulleys are of two kinds:—1. Fixed, which do not move out of their position. 2. Moveable, which rise and fall with the weight.

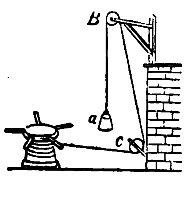
THE FIXED PULLEY.

Fig. 104.



When a pulley is fixed, as in fig. 104, two equal weights suspended to each end of a rope passing over it will balance each other, for if either of them be pulled down through any given space, the other will rise through an equal space in the same time, and the cord will be stretched uniformly throughout its length; and consequently as the velocities of both are equal, they must balance each other. Hence it appears that the fixed pulley gains no mechanical advantage: it is nevertheless attended with the great convenience of enabling the *direction* of the power to be varied; for by it a man may raise a weight to any point without moving from the place he is in, whereas otherwise he would have been obliged to ascend with the weight: it also enables several men together to apply their strength to the weight by means of the rope. Sometimes it becomes necessary to use two fixed pulleys, the power being applied by means of horse power, and often by the capstan. Thus, suppose it be required to raise a heavy weight *a*, fig. 105; two fixed pulleys *B* and *C* may be used, one end of the rope being attached to the weight; it is then carried over the pulley *B*, and returning downwards is

Fig. 105.



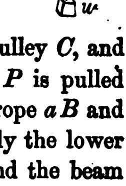
brought under *C*, and this end is fixed to the capstan, where the power is applied. By means of the fixed pulley a man may raise himself upwards or descend to any given depth; if he be seated in a chair or bucket, as represented in fig. 106, attached to one end of a cord, which is carried over a fixed pulley above, by pulling downwards the other side of the cord he may raise himself to a height equal to half its entire length; he may likewise descend to a depth equal to half the length of the cord: on this principle fire-escapes have been constructed, the pulley being hooked to some part of the building.

Fig. 106.

Moveable Pulleys.

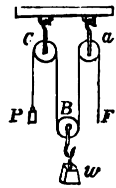
A *single moveable pulley* is represented in the annexed figure (107), and it differs from the *fixed pulley* by its hanging in the cord which passes under it, and from which the weight is suspended. A cord is carried from a fixed point *a*, and passing through the moveable pulley *B* attached to a weight *w*, passes over a fixed pulley *C*, and the power is applied at *P*. As the power *P* is pulled downwards, the length of the parts of the rope *aB* and *BC* will evidently be shortened, consequently the lower pulley must rise, the distance between it and the beam at the top becoming as much less as those two parts of the rope are made shorter; and as the weight *w* is sustained by the parts of the cords *aB* and *BC*, and these parts are equally stretched, each must sustain half the

Fig. 107.



weight: if therefore the power be equal to only half the weight, it will support it by the help of this machine.

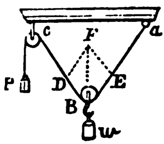
Fig. 108.



Let us now suppose that instead of the rope being fastened to a fixed point *a*, as in fig. 107, it be lengthened, and allowed to pass over a second fixed pulley, as in fig. 108, and hang down to *F*, so that a new force, equal to half the weight, on being applied at *F*, pulls the rope *a F* downwards; this new force will be equal to that by which the fixed point *a*, fig. 107, sustains its share of the weight: as by the law of *action and reaction* it will be equal to the force with which the weight acts upon it, and as the weight acts upon it with half its force, the force of the point *a* in sustaining the weight is equal to half the weight. But the new force at *F* acting over the fixed pulley *a* being equal to half the weight, it will consequently be equal to the force of the point *a*. Hence we conclude that, supposing this new pulley to be removed, the cords *aB* and *BC* are equally stretched by the actions of the weight and power and fixed point upon them; for the weight stretches each of them with a force equal to half its weight, and the power and fixed point on their side also stretch them equally with forces equal to half the weight: therefore, being stretched by equal forces, they must be equally stretched. The *halving of the weight* is therefore the mechanical advantage obtained by the moveable pulley; for example, if the weight *w* be 12 pounds, 6 pounds will be sustained at the fixed point *a*, fig. 107, and 6 pounds by the power *P*.

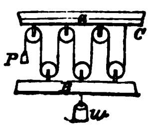
If, instead of the cords aB and BC hanging in a vertical direction, they hang obliquely, as in the annexed figure (109), a greater power than half the weight will be necessary to sustain it. In this case, to ascertain the amount of power required to support a given weight, we apply the principle of the composition and resolution of forces, as already explained at page 12: we must therefore draw a vertical line from B, consisting of as many inches as the weight consists of ounces. Suppose this line to extend to F, then from F draw FD parallel to aB, and FE parallel to CB; the amount of the weight represented by FB will be equivalent to two forces represented by DB, BE. The number of inches in BD will represent the number of ounces which are equivalent to the tension of the part BC of the cord, and in like manner the number of inches in BE will represent the number of ounces equivalent to the tension of the part of the cord Ba; and as the cord is equally stretched by the weight w, BD and BE must be equal, and the power P, which stretches the cord at PC, will be equal to each of them.

Fig. 109.



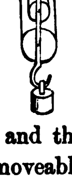
The mechanical power of pulleys may be almost indefinitely increased by combination. There are two different kinds of combinations or systems of pulleys; one consisting only of a single rope, and the other of several distinct ropes. The annexed figure (110) represents a single cord passing over several fixed pulleys. Suppose there be three pulleys fixed to the upper beam a, and three to the

Fig. 110.



lower beam *B*; let one end of a cord be fixed at *C*, and passing under the pulleys in the lower beam *B*, and over the pulleys in the upper beam *a*; let a power *P* be attached to the other end of the cord; then if a weight *w* be attached to the lower beam, and all the cords are parallel, every part of the cord sustains a pressure equal to the power *P*. In this case, as there are three pulleys attached to the lower beam from which the weight is suspended, the weight may be considered as divided into three equal parts, each pulley bearing a third of the weight; but since there are two cords to each lower pulley, the action of *each* of these three parts may be considered as again divided into two other equal parts,—therefore every cord will sustain one-sixth part of the weight, or 1 pound at *P* will balance 6 pounds at *w*.

Fig. 111.



In fig. 111 the number of cords sustaining the weight is four, and therefore the weight may be four times as great as the power; it is evident that each cord, as in the preceding illustration, sustains an equal part of the weight. The weight may therefore be considered as divided into four parts, each part being sustained by one cord. In calculating the expenditure of power, or diminution of weight in this machine, we have only to multiply the number of moveable pulleys by two, and the product will be the required power. Two moveable pulleys multiplied by two cords give a product of four; therefore the power to be exerted will be a fourth of the weight, and so on.

Fig. 112.

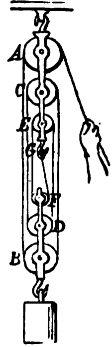
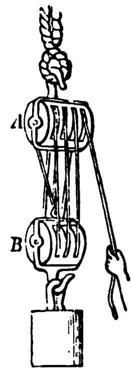


Fig. 112 is another representation of a system of pulleys having a single rope. The weight is attached to the lower block which is moveable and contains three wheels *B*, *D*, and *F*. The upper block is fixed and contains three wheels *A*, *C*, and *E*, and the rope attached to the hook *G* is successively passed over the wheels above and below, and after passing over the last wheel above, is attached to the power. The weight is sustained by all those parts of the cord which pass from the lower block; and as the force which stretches them all is the same, viz. that of the power, the effect of the weight must be equally distributed among them. It must therefore be evident that the weight will be as many times greater than the power as the number of cords which support the lower block. Thus, if there be six cords, as in the last example, each cord will support a sixth part of the weight.

Fig. 113.

The same principle equally applies to the annexed figures, 113 and 114, which are representations of a system of pulleys chiefly used on board of ships for raising and lowering sails and masts. This system is much more convenient for practical purposes than that represented in the preceding illustration, in which case the length of the two blocks renders it impossible to raise the weight to within a considerable distance of the point to which the system is suspended. In the accompanying diagram

Fig. 114.



the pulleys are placed side by side, instead of being arranged beneath one another, as in figs. 111 and 112. In the adjoining illustration (fig. 114) three moveable pulleys are inclosed in the block *B*, and three fixed pulleys are inclosed in the block *A*. If the weight be 1,200 pounds, it may be considered as in the former instances to be distributed over the six cords, and thus requiring only a power of 200 pounds to support it. This system of pulleys is, however, attended with the inconvenience of the ropes acting obliquely on the pulleys, which tends to increase their friction and to wear their axes; and in all cases where the power is applied obliquely, as was shewn in fig. 109, there will be a loss of power in proportion as the line of tension departs from the vertical. In calculating the power of systems of pulleys of this class, the weight of the lower block must always be considered as a part of the weight to be raised.

A system of pulleys has been contrived by the celebrated Smeaton, in which there are ten wheels in each block, arranged in two rows beneath one another. A single cord is made to pass over the pulleys in the order in which they are marked by the figures 1, 2, 3, &c. fig. 115. The tension of the cords being the same throughout, each cord acts upon the weight with a force equal to the power. Thus, a power of 20 will sustain a weight of 400, there being 10 pulleys in the lower block, and two cords to each pulley, $10 \times 2 = 20$, which being multiplied again by the power 20, produces 400.

In the system of pulleys already described, every pulley has a separate axle; and *each* pulley turning on its axle, must consequently be attended with friction.

Fig. 115.

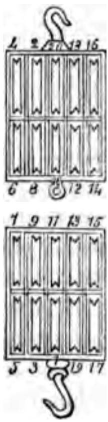
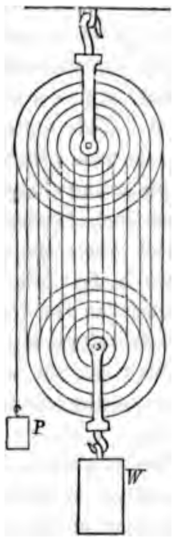


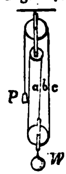
Fig. 116.



To obviate this defect, an ingenious contrivance has been suggested by White, by which all the pulleys on each block turn upon the *same* axis. Fig. 116 is a representation of White's pulley, in which there are two circular blocks consisting of pulleys placed upon one another, and revolving upon a common axis. The same cord is passed round each pulley in succession, commencing with the largest of the higher block, and terminating at the centre of the lower block, where it is fastened. For a fuller explanation of this pulley, as well as its mechanical efficacy, the reader is referred to the "Treatise on Mechanics," p. 35, in the Library of Useful Knowledge.

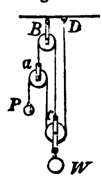
A single rope may be so arranged by means of one moveable pulley as to support a weight equal to three times the power. The annexed figure (117) represents this arrangement, where the power *P* of one pound supports the weight *W* of 3 pounds by means of the three parts of the cord *a*, *b*, and *c*. For each of the strings *a*, *b*, and *c*, supports the same tension, namely, a force equal to the weight of *P*; and the tension of them all is counteracted by the weight of *W*.

Fig. 117.



A weight may be made to support another weight four or five times as great as itself by means of two cords and two moveable pulleys, called *Spanish Bartons*, as represented in fig. 118. The two moveable pulleys have their sheafs attached by the same cord, *aBc*, passing over the fixed pulley *B*. The power *P* is made to act upon a second cord, passing over the first pulley under the third, and fixed to the beam *D*. In this case, each of the cords *Pa*, *ac*, and *cD*, supports a pressure equal to the power of *P*; and each of the strings *aB*, *Bc*, supports a pressure equal to twice the power *P*; and as the weight *W* counteracts the pressure upon the three cords *Dc*, *cB*, *ca*, it must therefore be four times as great as *P*.

Fig. 118.



A number of moveable pulleys may be combined so as to increase the power of the system to any extent. In fig. 119, a system of pulleys is represented with three ropes, in which the weight is 8 times the power. At the extremity of the first rope, a power of 4 pounds is suspended: this rope, marked 1, is passed under the moveable pulley *A*, and it is evident that it will sustain 8 pounds, 4 pounds being supported by each rope. The next rope, marked 2, passing under the moveable pulley *B*, in like manner supports 16 pounds, or 8 pounds on each side. The remaining cord, marked 3, passing under the moveable pulley *C*, supports 32 pounds, or 16 pounds on each side. Thus, 4 pounds at *P* will support 32 pounds at *W*. If, instead of 3 ropes and 3 moveable pulleys, there be 4 of each, then 4 pounds at *P* would support 64 pounds at *W*; and so on in the same proportion, as it is obvious that each rope which is added to such a system will double its effect. The power of this system may be greatly increased by substituting small fixed pulleys for hooks, as in fig. 120. In this case, the rope, instead of being attached to a hook, passes over the fixed pulley, and is fastened to the moveable pulley. Each moveable pulley, therefore, instead of being sustained by the equal tensions of two cords, is sustained by the equal tensions of three, so that the tension of the second rope is three times that of the first, which is equal to the power.

Fig. 119.

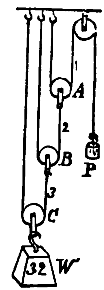
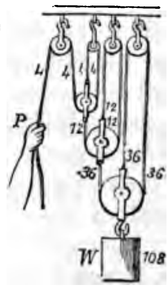
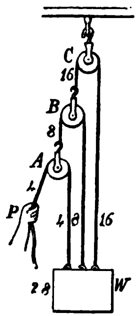


Fig. 120.



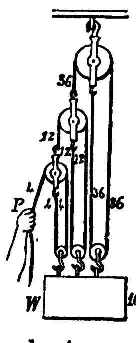
The tension of the third is three times that of the second, or nine times that of the first, and so on, the weight being three times the tension of the last rope; therefore 4 pounds at *P* will support 108 pounds at *W*.

Fig. 121.



An arrangement of pulleys is represented in the annexed figure (121), by which each rope, instead of being finally attached to a fixed point, as in fig. 120, is attached to the weight itself. *AB* are two moveable pulleys, and *C* a fixed pulley; a rope passing over the fixed pulley *C* is attached by one of its extremities to a bar bearing the weight *W*, and by the other to the sheaf of the moveable pulley *B*, over which passes a second cord attached in a similar manner to the bar, and carrying a third pulley *A*. The weight is in this case supported by three ropes; one stretched with a force equal to the power, another with a force equal to twice the power, and a third with a force equal to four times the power. The weight is therefore in this case seven times the power, or 4 pounds at *P* would support 28 pounds at *W*.

Fig. 122.

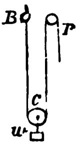


Such a system may be rendered much more powerful if the ropes, instead of being attached to the bar which sustains the weight, as in fig. 121, be made to pass through wheels, as in the annexed figure (122), and be finally attached to the pulleys above. In the present example, 4 pounds at *P* will support 104 at *W*, the weight being 26 times the power.

In considering these various combinations of pulleys, we have not taken into account the weights of the pulleys themselves. It is necessary, however, to observe, that in the two last examples, figs. 121 and 122, the weight of the pulleys *assists* the power in supporting the weight; and in figs. 119 and 120, the weight of the pulleys acts *against* the power. In the system called the Spanish Bartons, fig. 118, the weights of the pulleys to a certain extent neutralise each other.

It will in all cases be found that the *pulley*, like the lever and all other machines, obeys the principle of *virtual velocities*. For instance, let one of the ends of a rope be fastened to a hook *B* (fig. 123); if then we suppose the rope to be passed under a moveable pulley *C*, to which a weight *W* is attached, and the power to be applied at the other end of the rope *P*, it is clear that, in order to raise the weight *W* 1 foot, each rope sustaining the pulley together with the weight, must be shortened 1 foot,—that is, to raise the weight 1 foot, the power must move through 2 feet, and therefore the velocity of the power is twice that of the weight. In the same manner it may be proved that in fig. 107, where a power of 1 supports a weight of 3, if the power descends through 3 feet, the rope which is attached to the pulley supporting the weight must be shortened 3 feet, and therefore each of the three parts of the rope attached to the pulley will be shortened by 1 foot. In this case the velocity of the power is three times that of the weight. The same principle is equally applicable to all the sys-

Fig. 123.



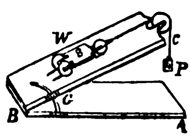
tems of pulleys above mentioned. In other words, when the power and weight balance each other in any system of pulleys, the power multiplied by the space through which it moves will be equal to the weight multiplied by the space through which it moves. This mechanical advantage which the pulley appears in theory to possess is much diminished in practice, as considerable allowance must be made for the friction of the cord, and of the pivots or axes on which the pulley turns: in most cases it is calculated that no less than two-thirds of the power is lost. *Friction*, however, in any system of pulleys may be considerably diminished by adopting an ingenious contrivance of Mr Garnet, called *friction rollers.*

CHAPTER X.

THE INCLINED PLANE.

THE inclined plane is the most simple of the mechanical powers. It consists of a plane surface supposed to be perfectly hard, forming some angle with a horizontal plane; the inclination of the plane may be to any extent, from that of a slight rise of the horizontal to almost an upright or perpendicular ascent. When a man has occasion to place a heavy barrel in a cart, if the barrel be too heavy for him to lift, he makes use of the inclined plane, which in this case would be a stout plank, one end resting on the cart and the other on the ground; the barrel is then pushed before him up the plank and into the cart. It is evident that the shorter this inclined plane is, the steeper is the ascent; and the longer it is, the ascent must be easier. We know from experience that it is much easier to push a rolling weight up a hill that rises gently, than up a hill that is very steep. Suppose the barrel to weigh 500 pounds: if no machinery is used, it will require a power equal to 500 pounds to raise it into the cart; but if it be rolled up the inclined plane, the power required will be less than 500 pounds, and the diminution of power depends on the smallness of elevation in the inclined plane. But this saving of power, as in all other instances of mechanical advantage, is accomplished only by a corresponding loss of time. The advantage gained by this mechanical power is just as much as the *length* of the plane exceeds its perpendicular *height*. In the annexed figure (124) let *AB* represent a horizontal plane, and *Bc* another plane in-

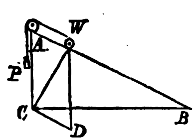
Fig. 124.



clined to it; let *ABc* be its angle of elevation, and *W* a weight placed upon it. If the plane be twice as long as it is high,—that is, if the length of the plane *Bc* be double the height from *A* to *c*,—then 4 pounds at *P* will balance 8 pounds anywhere between *B* and *c*. It is evident that if the plane *Bc* were lengthened, and the height *A* to *c* remain the same, that a less power than 4 pounds at *P* would sustain a weight of 8 pounds anywhere between *B* and *c*. From this explanation it may easily be inferred, that the less the elevation of the plane is, the less will be the power necessary to sustain a given weight upon it. In the same figure the inclined plane *Bc* is supposed to turn upon a hinge at *B*; it may therefore be heightened or depressed so that its angle of elevation may be increased or diminished, which is shewn by the graduated arch *G*.

This property of the inclined plane will be more readily understood by an application of the principle of the composition and resolution of forces. In the annexed figure (125) let *P* be the power acting in the direction *WA*, parallel to the inclined plane, and sustaining a weight *W*; draw *CW* perpendicular to *AB*, *WD* parallel to *AC*, and *CD* parallel to the inclined plane *AB*. Now the forces acting on *W* and keeping it in equilibrium are the power *P* in the direction *WA*, the weight of the body in the vertical direction *WD*, and the action of the plane in the direction *CW* perpendicular to the plane. Now it may be readily seen from

Fig. 125.



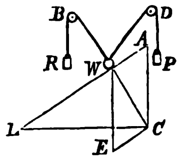
the demonstration of the parallelogram of force, that if three forces acting in directions parallel to the sides of a triangle keep a body in equilibrium, these forces are proportional to the sides of the triangle. The power P will therefore be in the same proportion to the weight W as WA is to WD, or as WA is to AC, or as AC is to AB. Thus, if the height of the plane be 1 foot, and its length 20 feet, a force of 1 pound will sustain a weight upon it of 20 pounds.

When it is said that the weight is sustained by the power, it must not be inferred that the *whole* weight presses upon the power, but only that the power keeps the weight from rolling down the plane. The entire weight is sustained partly by the power, and partly by the plane; and the power is relieved of as much of the weight as is sustained by the plane. The less the angle of inclination of the plane is, the less weight is sustained by the power, and the more by the plane; and so on the contrary.

When a body is rolled up an inclined plane, friction, as well as the gravity of the body, has to be overcome; the latter giving it a tendency to roll down to the lowest level, the plane hindering it from descending in a direct line to the earth. We have already observed in treating of gravity, p. 38, that a body descending to the earth by the force of its own gravity falls about $16\frac{1}{12}$ feet in the first second; but if it be rolled down an inclined plane, the number of feet it will roll down in the first second will be equal to the number of feet of inclination in $16\frac{1}{12}$ feet: thus if the inclination be 3 feet in $16\frac{1}{12}$, the body will roll down 3 feet in the first second, and so on.

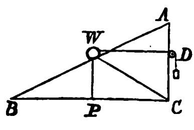
If the power, instead of acting in a direction *parallel* to the inclined plane, acts in an *oblique* direction, as *WD*, fig. 126, the proportion of the power to the weight may be found in a similar manner to that in fig. 125, by drawing *WC* perpendicular to the plane *AL*, *WE* perpendicular to the horizon, and *EC* parallel to *WD*; for the two forces represented by *WD*, *WE*, will be proportional to the forces *P*, *W* respectively, and these two forces may be compounded into the diagonal *WC*, which represents the pressure on the plane.

Fig. 126.



If we suppose the plane removed, and another power applied in the direction *WB*, this power represented by *R*, if it keep the body *W* at rest, will represent the reaction of the plane. In this case the body *W* is kept at rest by three forces; 1st, the force of gravity *W* acting in the vertical direction *WE*; 2nd, the power *P* acting in the direction *WD*; and 3rd, the resistance *R* of the plane, acting in the direction *WB* perpendicular to the plane. If the power act in a horizontal direction, or parallel to the base of the plane, its proportion to the weight will be that of the height of the plane to the base. In the following figure (127)* let *W* be the weight, and *D* the power, acting in the direction *WD* parallel to the base; the proportion of the power to the weight may be found in the same manner as in the preceding example. On this principle it will be seen that the power *D* will be to the weight *W* as *WD* is to

Fig. 127.



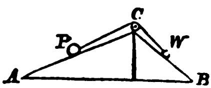
* In this figure the line *CW* should have been drawn *perpendicular* to *BA*—its present position is incorrect.

WP, or as *AC* is to *BC*, the *height* of the plane to its *base*.

From these examples it will be readily seen, that when the power is in a direction parallel to the plane (as in fig. 125), it acts to the greatest advantage; for if the power act in an *oblique* direction, as in fig. 126, it is evident that a part of the power is employed in lifting the weight from the plane; if it act in a direction *below* the plane (parallel to the base), as in fig. 127, a part of the power is employed in pressing the weight against the plane; but if it act in a direction *parallel* to the plane, the *whole* effect of the power is employed in drawing the weight *up* the plane; which is not the case when acting in a direction either *above* or *below* the plane, a part only being employed.

Sometimes a weight upon one inclined plane is raised or supported by another weight upon another inclined plane; in this case their proportion will be that of the lengths of the planes on which they rest. Fig. 128 represents two inclined planes of the same height, but different inclinations:

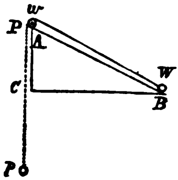
Fig. 128.



W and *P* are two weights resting on the planes, and connected by a cord passing over a pulley *C*. If the length of the longer plane from *A* to *C* be 2 feet, and the length of the shorter plane from *B* to *C* be 1 foot, then 4 pounds at *W* on the short plane will balance 8 pounds at *P* on the long one, and so on in the same proportion. This method of moving loads on two adjoining inclined planes is frequently applied in great public works, where various sloping rail-roads are used: a loaded waggon descends one inclined plane, and is made to draw up another waggon either empty or loaded on the other.

The principle of virtual velocities may be easily applied to the inclined plane. It will be seen from the following example, that if two forces balance each other on an inclined plane, and the whole be put in motion, the power multiplied by the space through which it is moved is equal to the weight multiplied by the space through which it ascends *vertically*. Let *AB*, fig. 129, be the inclined plane, *W* a weight at the foot *B* of the plane, and *P* the power at the top. Then if the power be pulled downwards until the weight arrives at the top of the plane, *W* will have been raised through a space equal to the *height* of the plane, while *P* will have moved through a space equal to the length of the rope which has passed over the pulley,—that is, the length of the plane. Hence *P* multiplied by the space through which it moves is equal to *W* multiplied by the space through which it ascends *vertically*. Thus, if the height of the plane be 1 foot, and its length 50 feet, the weight of *W* 50 pounds, and that of *P* 1 pound; then *P* will have to descend 50 feet while *W* is raised *vertically* through the space of 1 foot. In the inclined plane, therefore, as in every other mechanical power, velocity is lost in proportion as power is gained.

Fig. 129.



The inclination of a road is estimated by the height corresponding to some given length; thus a road is said to rise 1 foot in 20, when 20 feet of the road are taken as the length of an inclined plane, and the corresponding height 1 foot. A horse in drawing a cart up a hill, fig. 130, which rises 1 foot in 20, has the advantage of pulling only one-twentieth of the weight; for, although the cart is pulled to a distance of 20 feet, it is raised *upwards* only 1 foot.

Fig. 130.

If a plane be 64 feet in height, and 3 times 64, or 192 feet in length, a marble will roll down it in 6 seconds; for by the attraction of gravity it will fall to the earth in 2 seconds, because 16 feet (the space through which it will fall in the first second) multiplied by the square of 2 seconds, or 4, produce a product of 64, the height of the plane; but as the plane is 3 times as long as it is high, it must be 3 times as many seconds in rolling down the plane,—that is, 6 seconds*.

The power of inclined planes or hills may therefore in all cases be found by a simple rule of proportion; as for example, if it is desired to know what force will balance a weight of 375 pounds upon an inclined plane which rises 6 feet in 15 feet of its length, the proportion will stand thus—as 15 feet : 6 feet :: 375 pounds to a fourth term 150 pounds, which when found will be the answer required. To calculate what force the above weight of 375 pounds will balance or support upon a similar inclined plane, the proportion must be reversed, thus, as 6 feet : 15 feet :: 150 pounds to 375 pounds; and from

* In the above example the effect of *rotation* upon bodies descending an inclined plane is not taken into consideration. The times of actual descent vary according to the different shapes of bodies, as cylinders or spheres, and whether they be solid or hollow; but this is a subject of too intricate a nature to be treated of in this elementary work. The advanced student is referred to Whewell's "*Mechanics*," and Poisson, "Mécanique."

the proportion that holds good in all triangles of similar shape, it matters not how small a portion of the hill or inclined plane be taken to examine its power, provided its surface be an even plane; for the line *WP*, fig. 127, will be the half of the length *WB*, in the same way that *AC* is half the length of the entire surface *AB*. Either a large or small portion of an inclined plane may therefore be taken to examine its power, provided the rise or elevation that occurs in that part *only* be taken into account; and this rise in actual hills is best ascertained by the levelling instrument.

A fixed inclined plane is often used in assisting the elevation of great weights by means of other machines: it is supposed that in all the edifices of remote antiquity, where great masses of stone were employed, as in the pyramids of Egypt and the Druidical temples of this country, these vast blocks were elevated on inclined planes of earth or of scaffolding, with the assistance of levers and rollers. When a hill is too steep to be readily ascended, the road is either made to wind round the ascent or to advance in a zig-zag direction. The strain upon a horse in drawing a cart up a steep hill may be considerably diminished by leading him from one side of the road to the other, and thus advancing up the hill in a zig-zag direction instead of leading him directly forward. Inclined planes are frequently used for drawing boats out of one canal into another, and for wheeling barrows to the higher stage of scaffolding. All stairs or flights of steps are inclined planes, broken into successive steps for the purpose of affording a safer footing than could be obtained if the plane were not so divided. It is also used to produce gradual *descent*, as in the case of a ship launched into or drawn from the water.

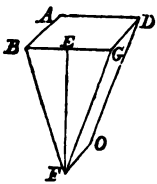
CHAPTER XI.

THE WEDGE.

Fig. 131.

The wedge is called, in mathematical language, a Triangular Prism: it consists of a solid body of wood, iron, or some other material, and is generally used in cleaving timber, in which case its edge is introduced into a cleft already made to receive it, as in fig. 131; and it is urged by the force of a hammer or mallet striking perpendicularly on its back. In the annexed figure (132) *DA* is the whole thickness of the wedge at its back *ABGD*, where the power is applied; *EF* is the depth or height of the wedge; *BF* the length of one of its sides; and *OF* is its sharp edge. It is evident that the wedge represented in fig. 131 will run more easily between any two bodies than that represented in fig. 132; but, on the other hand, it will make a much less separation than the other, which enters with more difficulty. The smaller wedge, fig. 131, would product a less effect, but would meet with less resistance; the larger one, fig. 132, would produce a greater effect, but would be resisted more. In calculating the mechanical efficacy of the wedge, the following rule is employed. The power is to each of the resistances as the width of the back is to one of its sides; but it is difficult to calculate the exact proportion of the effect to the resistance, as much depends on the force or number of blows which may be applied, and likewise the varied nature of the resistance; tough wood, such as oak, requiring more

Fig. 132.



force to split it than deal. A wedge of great inclination or obliquity would require considerable force to urge it forwards, for the same reason that a plane much inclined requires much force to roll a heavy body up it. If the angularity of the wedge were given, and the exact force of each blow ascertained, it would still be difficult to compute the power of the wedge in ordinary cases. In the splitting of timber and stone, for instance, the divided parts act as levers, and assist in opening a passage for the wedge, which circumstance necessarily increases its power. The *theory* of the wedge has not been here introduced, as it requires the aid of mathematical reasoning, and is quite inapplicable to practical purposes.

There is scarcely any instrument whose applications are more numerous than those of the wedge: chisels, nails, awls, needles, axes, sabres, &c. all act on the principle of the wedge. It is also used in a variety of cases where the other mechanical powers would be of no avail. This arises from its being driven principally by *impact;* the momentum of the blow is consequently much greater in comparison to the application of pressure to the lever. As an example of the enormous power of the wedge, it may be stated that the largest ships when in dock may be easily lifted up by driving wedges under their keels. It has sometimes happened that buildings,—such as a heavy chimney for a furnace,—have been found to incline, owing to the dampness of the foundation, and have been restored to their perpendicular position by wedges driven under one side. It is sometimes used in splitting rocks, which it would be impossible to effect by the lever, wheel and axle, or pulley; for the force of the blow or stroke shakes the cohering parts, and makes them separate more easily. In some parts of Derbyshire, where mill-stones are obtained from the silicious sand-rocks, wedges made of dry wood are driven into holes bored round the piece of rock intended to be separated from the mass: these wedges gradually swell by the moisture of the earth, and in a day or two lift up the mill-stone without breaking it. Builders, in raising their scaffolds, always tighten the ropes round their scaffolding-poles by means of wedges driven between the cords and the poles.

A knife may be considered as a wedge when employed in splitting; but if the edge be examined with a microscope, it is seen to be a fine saw, as is evident from the much greater effect all knives produce by being drawn along the materials against which they are applied, than what would have followed from a direct action of the edge.

It appears from the results of some experiments made in the dockyard at Portsmouth, on the comparative effect of driving and pressing in large iron and copper bolts, that a man of medium strength, striking with a mall weighing 18 pounds, and having a handle 44 inches in length, could start or drive a bolt about one-eighth of an inch at each blow, and that it required the direct pressure of 107 tons to press the same bolt through that space; but it was found that a small additional weight would press the bolt completely home*.

* Encyclopædia Metropolitana, "Mixed Sciences," vol. vi. p. 52.

CHAPTER XII.

THE SCREW.

THE screw cannot properly be called a simple machine, because it is seldom used without the application of a lever or winch to assist in turning it; and then it becomes a compound engine of very great force, either in pressing the parts of bodies closer together, or in raising great weights. The screw may be considered as a modification of the inclined plane, which may be seen by cutting a piece of paper *A*, fig. 133, into the form of an inclined plane and rolling it round the cylinder *D*; its edge *B* will then represent the spiral line called the *thread* of the screw. Fig. 134 represents the paper when wound tight round the cylinder. In the application of the screw, the weight or resistance is not, as in the inclined plane and wedge, placed upon the surface of the plane or thread. The power is transmitted by means of another screw, called the *nut*, through which it passes. The *nut* consists of a concave cylinder, on the interior surface of which a spiral groove is cut, which exactly fits the thread of the screw. In order that the effect of the power may be conveyed to the resistance by means of this machine, either the *nut* or the screw must be fixed. If the *nut* be fixed, the screw may be continually turned round by a lever inserted in one end of it till it reaches its extremity; and if the *screw* be fixed, the *nut* may be

Fig. 133.

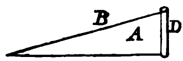
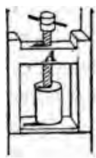


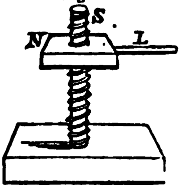
Fig. 134.

Fig. 135.



turned round it by means of a lever until it be moved from the bottom of the screw to the top. Fig. 135 represents the *nut A* to be fixed. If the screw be turned to the right, it will advance downwards, while the *nut* is stationary. Fig. 136 represents the *screw* to be fixed; while the nut *N*, by being turned by the lever *L* from left to right will advance down the screw. In calculating the advantage gained by the screw, there are two things to be taken into consideration, the circumference of the cylinder round which the screw is cut, and the distance between the threads of the screw. It is evident that the winch must turn the cylinder once round before the weight or resistance can be transmitted from one spiral winding to another; therefore, as much as the circumference of a circle described by the handle of the winch is greater than the interval or distance between the spirals, so much is the force of the screw.

Fig. 136.



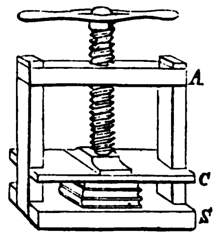
Hence it appears that the longer the winch is, and the nearer the spirals are to one another, so much the greater is the force of the screw; therefore, to increase the mechanical efficacy of the machine, we must either increase the length of the lever by which the power acts, or diminish the distance between the spirals. For instance, if there be two screws, the circumferences of whose cylinders are equal to one another, but one having the threads 1 inch apart, and the other 3 inches apart, it will readily be seen, on considering the principle of the inclined plane, that the screw having the threads only 1 inch apart will have three times the advantage of the screw whose threads are 3 inches apart. As we have already shewn that if the *height* of two inclined planes be the same, but the base of one three times greater than that of the other, the mechanical advantage gained by the longer base would be three times more than that gained by the other, but at the same time the process of rising to a given height would be slower; in applying this principle to the screw, it is obvious that the screw having threads 1 inch apart must be turned round three times as often as that having threads three inches apart, to go through the same space; therefore, as space is passed, or time lost in proportion to the advantage gained, we conclude that three times more advantage is gained by the former than by the latter screw.

The power of the *screw* only, without the application of the lever, may be found by the following rule:—"As the circumference of the screw is to the distance between the threads, so is the weight to the power;" but as this machine is seldom worked without the lever, the circumference which the outer end of the lever describes is taken instead of the circumference of the screw itself. In estimating the true effect of the screw in connection with the lever, we must multiply the circumference which the lever describes by the power. Thus: "The power multiplied by the circumference which it describes is equal to the weight or resistance multiplied by the distance between the two contiguous threads."

If then we know the length of the lever, the distance between the threads, and the weight to be raised, we can readily calculate the power; or, if it be required to know the amount of weight the screw will raise, we have only to ascertain the power, the distance of the threads, and the length of the lever. Let us now calculate the advantage gained by a screw, the threads of which are half an inch distant from one another, and the lever 6 feet long. If the radius of a circle be given, in order to find the circumference we must multiply the radius by 6,—the circumference is a little more than 6 times the radius; but as this will answer all common purposes, we will call it just 6 times. The lever being 6 feet in length, the circumference of the circle made by its revolution will be 6 feet multiplied by 6, which is 36 feet or 432 inches; but during this revolution the screw is raised only half an inch, therefore the space passed by the moving power will be 864 times greater than that gone through by the weight; consequently the advantage gained is 864, or 1 pound applied to the lever will balance 864 pounds acting against the screw. Hence it follows that there are two ways of increasing the mechanical advantage of the screw; either by increasing the length of the lever by which it is turned, or by diminishing the distance between the threads. Let us now suppose the threads of the screw so fine as to be only one quarter of an inch apart, and that the length of the lever be 10 feet, or 120 inches; the circumference of the circle made by this lever will be 10 × 6, which is 60 feet, or 720 inches, or 2,880 quarter inches; and as the elevation of the screw is but one quarter of an inch, the power will move through a space 2,880 times greater than that moved through by the weight; therefore a power of 1 pound acting at the end of the lever will raise 2,880 pounds. It is necessary, however, to observe, that the friction of the parts of a screw is so great, that in practice its effect is far less than the theory would lead us to expect, as a third of the amount of power is generally required to be added to overcome the friction of the machine.

Of all the mechanical powers, the screw is the best calculated to produce great pressure accompanied with continuous action, and a retention of the pressure: the action of the lever alters continually, and the pressure is intermittent; but the screw acts continually, with the same pressure, in the same direction, and always retains its hold.

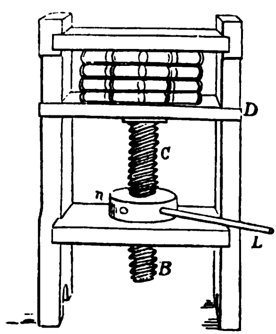
Fig. 137.



The annexed figure (137) is a representation of the standing press used by bookbinders for pressing their books, which is a good example of the screw when used to produce a great pressure.

The screw is turned by means of a lever fixed across the top; to the lower end of the screw is attached a pressing-table *C*; so that when the screw is turned in one direction, a pressure is exerted upon the books placed on the fixed plate *S*, which pressure may be immediately removed by turning the screw in the opposite direction. In this example the *nut* is fixed, being in the cross-beam *A*. In the following illustration (fig.

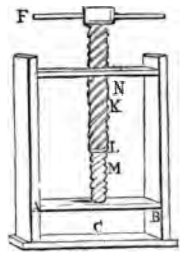
Fig. 138.



138) the *screw* is fixed, and the *nut* is moveable. The board *D*, moveable between the sides of the frame, is urged by the screw *CB*, which is capable of moving either upwards or downwards, but incapable of revolving. The nut *n*, worked by the lever *L*, is capable of revolving, but does not advance in the direction of the screw. Every complete revolution of the nut urges the screw upward through a space equal to the distance between two contiguous threads.

It has already been observed, that in the screw the weight which can be supported by a given power depends upon the proportion between the circumference which the power describes, and the distance between two contiguous threads of the screw. Hence it is evident that the mechanical advantage of the screw may be increased by lengthening the lever by which the power acts, or by cutting the threads sufficiently fine; but, although there is no limit in theory to the increase of this mechanical advantage by these means, yet it is often practically inconvenient to increase the length of the lever; for the space through which the power should act would be too great for practical purposes; and, on the other hand, if the threads of the screw be cut too fine, they become too weak to support the required pressure. To obviate this inconvenience, an ingenious contrivance has been invented by Mr. Hunter, similar in principle to the wheel and axle, p. 100, where one part of the axle is thicker than the other, so as to enable a very small force to sustain a very large one. This contrivance consists in the combination of two screws, one of which works within the other. The mechanical power of such a screw *does not depend* upon the actual distances between the *threads of the* two screws of which it is composed, but upon the *difference* of those distances. Hence, therefore, the threads may have any strength and magnitude, provided they do not greatly differ in thickness from each other. Fig. 139 is a representation of this machine. *KL* is the greater screw playing in the fixed nut *N*. This screw is hollowed out, the interior of which is a nut corresponding with an *external* screw cut upon the smaller cylinder *M*, and attached to the sliding press *B*. During every revolution of the screw, the cylinder *KL* descends through a space equal to the distance between its threads. At the same time the smaller cylinder *M* ascends through a space equal to the distance between the threads cut upon it; and the combined effect will be, that the smaller screw *M*, and the sliding board *B*, to which it is attached, will be moved *downwards* through a space equal to the difference of the distances between the threads of the two screws. If the threads of the two screws were perfectly equal, and the screw *KL* were turned round by a power applied to the lever *F*, the sliding board *B* would retain its position; for the larger screw would be moved *downwards* just as much as the smaller screw would be raised *upwards;* but if the distance between the threads in the smaller screw *M* is rather less than that in the larger screw *KL*, the sliding board *B* will be pressed downwards in each revolution of the lever *F* through a space equal to the difference of the distance between two threads in *KL* and two threads in *M*. It is clear, therefore, that the effect of this machine is the

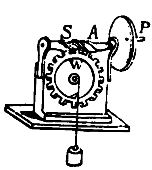
Fig. 139.



same as if a simple screw were used, in which the distance between the threads is equal to the difference of the distances between the threads of the two screws, and therefore the power will be to the weight as the difference of the distances between the threads of the screws is to the circumference described by the power at *F*. From the above illustration, it will readily be seen that the mechanical advantage of this machine is increased by diminishing the difference of the distance between the threads of the screws*.

Sometimes the thread of the screw, instead of urging forward the nut, is made to act upon the teeth of a wheel, as in fig. 140. The screw is in this case called an *endless screw*, because its action upon the wheel may be continued without limit. In this machine there is a combination of two mechanical powers—the screw, and the wheel and axle. Let *S* represent a screw cut upon a horizontal spindle *A*, working in the teeth of the wheel *W*, and let *P* be the winch to which the power is applied. The distance between any two threads of the screw must exactly correspond with the width of one of the teeth of the wheel; so that a complete revolution of the screw is necessary to move the circumference of the wheel through a distance equal to one only of its cogs. If the wheel *W* consists of sixteen teeth, it is evident that for every time the spindle *A*, and the screw *S*, are turned round by the winch *P*, the wheel *W* will be moved one tooth by the screw; and therefore, in sixteen

Fig. 140.



* "*Easy lessons in Mechanics.*" J. W. Parker.

revolutions of the winch, the wheel *W* will be turned once round.

The uses of the screw are innumerable. It is used in coining where the impression of a die is to be made upon a piece of metal. It is also employed in taking off copper-plate prints, and for printing in general. By its aid a large bale of cotton is condensed into a small package, and, from being the lightest and most buoyant of substances, becomes dense enough to sink in water. Sometimes buildings are raised from an inclined to a vertical position, by means of a small screw, acted upon by a comparatively small force. It is also of great utility in astronomical calculations, by affording an easy and very exact method of measuring or subdividing small spaces. An ordinary screw will divide an inch into five thousand parts; but the fine hardened steel screws which are applied to the limbs of astronomical instruments will go much further. In this case it is called a *Micrometer Screw*, from the Greek words Μικρὸς, little, and Μέτρον, a measure. The *gimlet* and *auger* are examples of the screw, both of which may be considered as an inclined plane wrapped round a *cone* instead of a *cylinder*. The power of these instruments is very much increased by their terminating in a point. When liquids or juices are to be expressed from vegetables or fruit, the screw is generally used. The cyder-press is an example of this machine so applied; and in all cases where great pressure is required, the power of the screw is often employed.

It is not unfrequently used in flour-mills for pushing the flour which comes from the mill-stones to the end of a long trough, from which it is conveyed to other parts of the machinery, in order to undergo the remaining process. In this case, the spiral threads are very large in proportion to the cylinder on which they are fixed.

A common corkscrew is the thread of the screw without the spindle, and is used, not to correct opposing forces, but merely to enter and fix itself in the cork. Complicated corkscrews are now made, which draw the cork by the action of a second screw, or of a toothed rod or rack and pinion.

CHAPTER XIII.

FRICTION.

We have already observed that in calculating the effects of mechanical contrivances a considerable allowance must be made for the friction of the moving parts on each other; but as the effect of friction on machinery is an important point of consideration, it has been thought necessary to treat of the subject in a separate chapter.

Friction is the effect produced by the surface of one body moving or tending to move upon the surface of another, for although the surfaces of bodies may appear smooth, there are nevertheless in all cases small asperities spread over them; and therefore when two surfaces are in contact, the small projections or roughnesses of the one fall into the cavities of the other, and thus tend to impede motion. The friction of the various parts of machines occasioned by roughness of the contiguous surfaces, is generally increased in time by the iron becoming rusty, the wood soft and rotten, and the ropes hard and stiff. Old hinges of doors, or window-shutters, which have not been stirred or opened for a long time, move stiffly on account of the friction occasioned by rust. In like manner, if we have occasion to open a rusty lock with a rusty key, we find it difficult to overcome the resistance the key meets with in turning through the wards. Screws of any kind that have been unused for a long time are generally turned with great difficulty.

The friction of one piece of iron, wood, brick, stone, &c. on another piece of the same substance has been measured by making the second piece an inclined plane, and then gradually lifting one end of it until the upper mass began to slide: the inclination of the plane, just before the sliding commences, is called the angle of repose.

The following means are used to diminish friction between rubbing surfaces, and either singly or in combination, according to circumstances.

1. Making the rubbing surfaces smooth; but this must be done within certain limits, for great smoothness allows the bodies to approach so near that a degree of cohesion takes place.

2. Letting the substances which are to rub on each other be of different kinds. Axles are made of steel, for instance, and the parts on which they bear are made of brass: in small machines, as time-keepers, the steel axles often play in agate or diamond. The swiftness of a skaiter depends much on the great dissimilarity between steel and ice.

3. Interposing some lubricating substance between the rubbing parts; as oils for the metals, soap, grease, black-lead, &c. for the wood. There is a laughable illustration of the tendency which soap or grease has to obviate friction in the holiday sport of soaping a lively pig's tail, and offering him as the prize of the clever fellow who can catch and hold him fast by his slippery appendix.

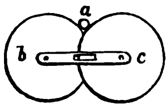
4. Diminishing the extent of the touching surfaces, as in making the rubbing axis of a wheel very small.

5. Using wheels, as in wheel carriages, instead of

dragging a load along the ground. Casters on household furniture are miniature wheels.

6. Using what are called friction wheels; which still further diminish the friction even of a smooth axis, by allowing it to rest on their circumferences, which turn with it. In the annexed figure (141), *a* represents the end of an axis, and *b* and *c* two friction wheels, on which it rests.

Fig. 141.



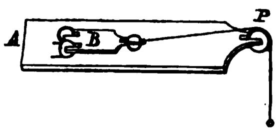
7. Placing the thing to be moved on rollers or balls, as when a log of wood is drawn along the ground upon rounded pieces of wood, or when a cannon, with a flat circular base to its carriage, turns round by rolling on cannon balls laid on a hard level bed. In these two cases there is hardly any friction, and the resistance is merely from the obstacles which the rollers or balls will have to pass over. Of all rubbing parts the joints of animals are those which have the least friction, considering the strength, frequency, and rapidity of their movements. We study and admire the perfection found in them, without being able very closely to imitate them.

Wheel carriages illustrate many of the circumstances connected with friction; but as a detailed explanation of them would be too extended for an elementary work like the present, the reader is referred to a very interesting chapter on the subject in Dr. Arnott's Elements of Physics, from which the greater part of the above remarks have been taken.

It will be readily inferred from the preceding observations, that the resistance from friction depends on the roughness of the surface and the force of the pressure. When the surfaces are the same, a double pressure will produce a double amount of friction; a treble pressure a treble amount of friction; and so on.

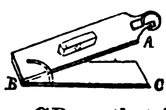
It has been found by experiment that the resistance arising from friction does not at all depend on the extent of the surface of contact; but if the nature of the surfaces and the amount of pressure remain the same, this resistance will be equal whether the surfaces which move one upon the other be great or small; for example, if the moving body be a flat block of wood, the face of which is equal to 4 inches square, and the edge to a quarter of an inch square, it will be subject to the same amount of friction, whether it move upon its broad face or upon its narrow edge. This will be obvious from the following calculation:—Let us suppose the weight of the block to be 4 ounces; when it rests upon its face, a pressure to this amount acts upon a surface of 16 square inches, so that a pressure of a quarter of an ounce acts upon each square inch. The total resistance arising from friction will therefore be 16 times that resistance which would be produced by a surface of 1 square inch under a pressure of a quarter of an ounce. Now, suppose the block placed upon its edge, there is then a pressure of 4 ounces,—that is, 16 quarter ounces upon a surface equal to a quarter of an inch square. But it has been already shewn, that when the surface is the same, the friction must increase in proportion to the pressure. Hence we infer that the friction produced in the present instance is 16 times the friction which would be produced by a pressure of a quarter of an ounce acting on one square inch of surface, which is the same resistance as that which the body was proved to be subject to when resting on its face.

Fig. 142.



The laws of friction may be illustrated by the following experiments:—Upon a horizontal plane *A*, fig. 142, let a small carriage *B* be placed, having a cord attached to it, and carried parallel to the plane over a wheel at *P*; a very trifling weight suspended to the end of the cord will be found sufficient to move the carriage along the plane. If, instead of the carriage, a block of wood be substituted of *the same weight*, and having a rough surface of green baize, it will be found that a greater weight will be required to move this along the plane than was required for the carriage. The weight in both cases is equivalent to the amount of friction. Again, let the weight of the block of wood be doubled by placing upon it a weight equal to its own weight. The pressure will now be doubled, and it will be found that the former weight suspended to the cord will be quite inadequate to overcome the friction; but if another weight be added just sufficient to overcome the friction as before, it will be seen that the whole weight necessary to produce this effect is exactly twice the weight which produced it in the former case. Thus it appears that a double amount of pressure produces a double amount of friction.

Fig. 143.



The laws of friction may be further illustrated by the aid of the inclined plane. Let the block of wood *W*, fig. 143, be placed upon a plane *AB*, which is hinged to an horizontal plane *CB*, so that it can be raised to any proposed elevation. Let the plane

AB be slowly raised until it acquires such an elevation that the force of the body down the plane is just sufficient to overcome the friction, and that the body will therefore commence to move. The tendency of the block *W* to descend upon *AB* will bear the same proportion to its entire weight as the perpendicular *AC* bears to the length of the plane *AB*. Thus, if the length *AB* be 12 inches, and the height *AC* be 3 inches,—that is, a fourth part of the length,—then the tendency of the weight to move down the plane is equal to a fourth part of its whole amount. If the weight were 12 ounces, and the surfaces perfectly smooth, a force of 3 ounces acting up the plane would be necessary to prevent the descent of the weight. It need scarcely be remarked that all the mechanical powers are subject to friction, and, as before observed, allowance must be made in calculating the effects produced by them; but some machines are necessarily subject to less friction than others, on account of the surfaces in contact being few.

The lever is subject to little friction, as it balances round a point in theory, and a very small surface in practice.

The pulley is subject to friction from the ropes passing over the blocks, and the axles turning about the centres; for the last reason it is thought necessary to have pulleys as large as convenient, because in a large pulley the rope acts at a greater distance from the centre where the friction is, and consequently has the greater tendency to overcome it.

The wheel is subject to friction by its pressing with all its own weight and that of the weight it sustains upon its supports, although this inconvenience may be much lessened by proper contrivances.

In the inclined plane great allowance must be made for the effect of friction, which must materially modify any calculation as to the advantage it affords.

The wedge and screw are both subject to a great deal of friction; the moving surfaces being contiguous, more care is required in keeping these machines in order.

EXPLANATION OF SCIENTIFIC TERMS.

ACCELERATION, the increase of velocity in moving bodies. When this increase is equal in equal times, it is called uniformly accelerated motion.

ANGLE, the inclination of two lines meeting together in a point. Angles in Geometry are called *right*, *acute*, and *obtuse*. A *right* angle contains 90 degrees; *acute* angles less, and *obtuse* angles more than 90 degrees.

ATTRACTION OF GRAVITATION, see GRAVITY.

BALANCE, a lever turning on a pivot or fulcrum, and used for the purpose of weighing different bodies.

CENTRE OF GRAVITY, that point in a body from which if the body could be suspended, all the parts of the body would, in any situation, balance each other.

CENTRIFUGAL FORCE is that force by which a body in moving round a centre tends to recede from that centre.

CENTRIPETAL FORCE is that force by which a body tends towards some point as a centre.

CIRCUMFERENCE, the curve line which bounds a circle. The circumference of every circle is supposed to be divided into 360 parts, called degrees.

COMPOSITION AND RESOLUTION OF FORCES: when we substitute for a single force two or more forces, of which it is the resultant, the process is called the *Resolution of force*, and the contrary process the *Composition of force*.

CONE, a solid figure, having a circle for its base, and its top terminated in a point or vertex.

DIAGONAL, a right line drawn across a quadrilateral figure from one angle to another.

DIAMETER, a right line passing through the centre of a circle, and terminated at each side by the circumference.

DYNAMICS, that division of the science of Mechanics which treats of the motion of bodies.

EQUILIBRIUM: when two or more forces acting in a body keep that body at rest, these forces are said to be *in equilibrio*, which signifies *equally balanced*.

FORCE is the name of any exertion which produces or tends to produce a change in the state of a body, either by moving that body when at rest, or by stopping or changing its progress if already in motion.

FRICTION, the rubbing of the surfaces of bodies upon one another. In calculating the effects of machinery, allowance must always be made for friction.

FULCRUM, the prop or support by which a lever is sustained.

GRAVITY is a term given to that tendency which all bodies have to fall to the centre of the earth.

HORIZONTAL, any thing which is on a level with, or parallel to, the horizon. Thus we say a horizontal plane.

IMPULSE, the direct action of one body upon another in the production of motion.

INERTIA, or Inactivity, is that property of matter which causes it to continue in the same state, either of rest or of uniform motion in a right line, unless changed by some external force.

MATTER is the general name of every substance that has length, breadth, and thickness.

MOMENT: the product of a force and the distance of its direction from a given point is called *the moment of the force with respect to that point.*

MOMENTUM, the impetus or force of a moving body, which is always equal to the quantity of matter multiplied into the velocity.

MOTION is the moving of a body, or any parts of a body, from one place to another.

OSCILLATION is a term applied to the vibration of a pendulum.

PARABOLA, a figure arising from the section of a cone when cut by a planc parallel to one of its sides.

PARALLEL LINES: when two straight lines in the same plane are everywhere equidistant from each other, they are said to be parallel.

PARALLELOGRAM, a four-sided right-lined figure, whose opposite sides are parallel to each other.

PERCUSSION, a shock or stroke given by a moving body.

PERPENDICULAR, a line falling directly on another line, so as to make equal angles on each side.

POWER denotes any force, which, being applied to a machine, tends to produce motion.

RADIUS, a right line drawn from the centre to the circumference of a circle.

RESISTANCE denotes in general any power which acts in an opposite direction to another, so as to destroy or lessen its effects.

RESULTANT FORCE is the force which arises from combining or compounding two or more forces into one.

STATICS is that division of the science of Mechanics which treats of the powers which preserve bodies in a state of rest.

TRIANGLE, a figure of three sides and three angles.

VELOCITY is the rate at which a body moves, and is reckoned by the number of units of length passed over in a unit of time.

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